Deterministic Incremental Dependency Parsing

Animated illustration of algorithms presented in Nivre, 2008

Jacob Louis Hoover

April, 2021

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- 1.1 Dependency structures
- 1.2 Incremental dependency parsing
- 2. Stack-based Algorithms
- 2.1 Arc-standard stack-based algorithm
- 2.2 Arc-eager stack-based algorithm
- 3. List-based Algorithms
- 3.1 Non-projective list-based algorithm
- 3.2 Projective list-based algorithm
- 4. Experimental Evaluation

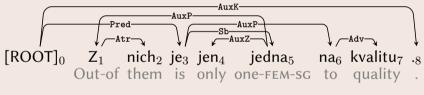
1 Preliminaries

Dependency structures:

Examples

■ A projective English dependency tree from the Penn Treebank (converted to dependency parse with Nivre's Penn2Malt).

■ A non-projective Czech dependency tree from the Prague Dependency Treebank.



("Only one of them concerns quality.")

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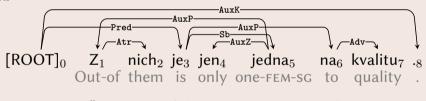
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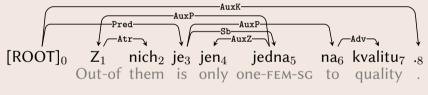
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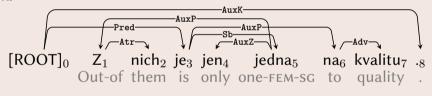
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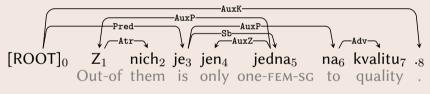
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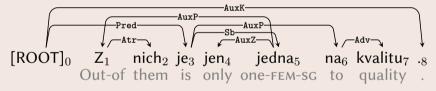
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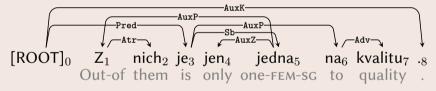
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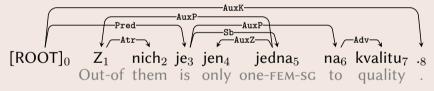
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 - by properties Root, Single-Head, and Acyclicity
- define property projectivity
 - there exists a path from the head of an arc to any node inside the span of the arc

Incremental dependency parsing

Common formalization incremental dependency parsing algorithms:

■ transition system $S = (C, T, c_s, C_t)$

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$\mathsf{Parse}(x = (w_0, \dots, w_n))$

- 1: $c \leftarrow c_s(x)$
- 2: while $c \notin C_t$ do
- $c \leftarrow [o(c)](c)$
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- Stack-based Algorithms (for projective structures)
 - arc-standard
 - arc-eager
- List-based Algorithms
 - non-projective
 - projective

Arc-eager stack-based algorithm

Stack-based Algorithms

Definition

A stack-based configuration for a sentence $x = (w_0, w_1, \dots, w_n)$ is a triple $c = (\sigma, \beta, A)$, where

- 1. σ is a stack of tokens $i \leq k$ (for some $k \leq n$) \leftarrow will represent as a list with head to right
- 2. β is a buffer of tokens j > k, \leftarrow will represent as a list with head to left
- 3. A is a set of dependency arcs such that $G = (\{0, 1, \dots, n\}, A)$ is a dependency graph for x.

Definition

A stack-based transition system is a quadruple $S = (C, T, c_{\text{start}}, C_{\text{terminal}})$, where

- 1. *C* is the set of all stack-based configurations,
- 2. $c_{\text{start}}(x = (w_0, w_1, \dots, w_n)) = ([0], [1, \dots, n], \varnothing),$
- 3. T is a set of transitions, each of which is a function $t: C \to C$,
- **4.** $C_{\text{terminal}} = \{ c \in C \mid c = (\sigma, [], A) \}.$

Transitions LEFT-ARC $_l$ $(\sigma|i,j|\beta,A) \Rightarrow (\sigma,j|\beta,A \cup \{(j,l,i)\})$ RIGHT-ARC $_l^s$ $(\sigma|i,j|\beta,A) \Rightarrow (\sigma,i|\beta,A \cup \{(i,l,j)\})$ SHIFT $(\sigma,i|\beta,A) \Rightarrow (\sigma|i,\beta,A)$ Preconditions LEFT-ARC $_l$ $\neg[i=0]$ $\neg\exists k\exists l'[(k,l',i) \in A]$ RIGHT-ARC $_l^s$ $\neg\exists k\exists l'[(k,l',j) \in A]$

Figure: Transitions for the arc-standard stack-based parsing algorithm.

"

The arc-standard parser is the closest correspondent to the familiar shift-reduce parser for context-free grammars (Aho, Sethi, and Ullman 1986).

The Left-Arc $_l$ and Right-Arc $_l^s$ transitions correspond to reduce actions, replacing a head-dependent structure with its head, whereas the Shift transition is exactly the same as the shift action.

One peculiarity of the transitions, as defined here, is that the "reduce" transitions apply to one node on the stack and one node in the buffer, rather than two nodes on the stack. The reason for this formulation is to facilitate comparison with the arc-eager parser and to simplify the definition of terminal configurations.

Arc-eager stack-based algorithm

Illustration Arc-standard transition sequence for English example sentence:

| $[ROOT]_0$ | Economic ₁ | $news_2$ | had_3 | little ₄ | effect ₅ | on_6 | financial ₇ | markets ₈ | .9 |
|------------|-----------------------|----------|---------|---------------------|---------------------|--------|------------------------|----------------------|----|
|------------|-----------------------|----------|---------|---------------------|---------------------|--------|------------------------|----------------------|----|

| | σ | β | A | |
|----|----------|----------------|------|---|
| 1. | ([0], | $[1,\ldots,9]$ |], Ø |) |

Arc-eager stack-based algorithm

Illustration Arc-standard transition sequence for English example sentence:

[ROOT]₀ Economic₁ news₂ had₃ little₄ effect₅ on₆ financial₇ markets₈ .₉

| | | σ | $oldsymbol{eta}$ | A |
|----|-------------------------|----------|-------------------------------------|-----|
| 1. | Shift \Longrightarrow | ([0], | $[1, \ldots, 9],$ $[2, \ldots, 9],$ | Ø) |

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| | | | σ | β | A | |
|----|----------------------------------|---|---------|-------------------|--------------------------|---|
| 1. | | (| [0], | $[1,\ldots,9],$ | Ø |) |
| 2. | $SHIFT \implies$ | (| [0, 1], | $[2,\ldots,9],$ | |) |
| 3. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0], | $[2, \ldots, 9],$ | $A_1 = \{(2, NMOD, 1)\}$ |) |

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Illustration Arc-standard transition sequence for English example sentence:

__NMOD____SBJ__ [ROOT]₀ Economic₁ news₂ had₃ little₄ effect₅ on₆ financial₇ markets₈ .₉

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| 5. | $Left	ext{-}Arc_{\mathtt{SBJ}} \implies$ | (| [0], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3, \mathtt{SBJ}, 2)\}$ |) |

Arc-eager stack-based algorithm

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| 9. | $SHIFT \implies$ | (| [0, 3, 5], | $[6,\ldots,9],$ | A_3 |) |

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| 6. | Shift \Longrightarrow | (| [0, 3], | $[4,\ldots,9],$ | A_2) |
| 7. | Shift \Longrightarrow | (| [0, 3, 4], | $[5,\ldots,9],$ | A_2 |
| 8. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0, 3], | $[5,\ldots,9],$ | $A_3 = A_2 \cup \{(5, \texttt{NMOD}, 4)\})$ |
| 9. | Shift \Longrightarrow | (| [0, 3, 5], | $[6,\ldots,9],$ | A_3 |
| 10. | Shift \Longrightarrow | (| [0, 3, 5, 6], | [7, 8, 9], | A_3 |

Arc-eager stack-based algorithm

Illustration Arc-standard transition sequence for English example sentence:

| | | | σ | β | A |
|-----|-------------------------------------------|---|------------------|-----------------|-----------------------------------------------|
| 1. | | (| [0], | $[1,\ldots,9],$ | Ø) |
| 2. | $SHIFT \implies$ | (| [0,1], | $[2,\ldots,9],$ | \varnothing |
| 3. | $Left	ext{-}Arc_{\mathtt{NMOD}} \implies$ | (| [0], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\} $ |
| 4. | $SHIFT \implies$ | (| [0, 2], | $[3,\ldots,9],$ | A_1) |
| 5. | $Left\text{-}Arc_{\mathtt{SBJ}} \implies$ | (| [0], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3, SBJ, 2)\} $ |
| 6. | $SHIFT \implies$ | (| [0, 3], | $[4,\ldots,9],$ | A_2) |
| 7. | $SHIFT \implies$ | (| [0, 3, 4], | $[5,\ldots,9],$ | A_2) |
| 8. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0, 3], | $[5,\ldots,9],$ | $A_3 = A_2 \cup \{(5, \texttt{NMOD}, 4)\})$ |
| 9. | $SHIFT \implies$ | (| [0, 3, 5], | $[6,\ldots,9],$ | A_3 |
| 10. | $SHIFT \implies$ | (| [0, 3, 5, 6], | [7, 8, 9], | A_3 |
| 11. | $SHIFT \implies$ | (| [0, 3, 5, 6, 7], | [8, 9], | A_3 |

Arc-eager stack-based algorithm

| | ✓—NMOD—— | ✓ SBJ ✓ | -NMOD-\ | NMOD—— |
|------|--------------------------------------------|------------------------|------------------------------------|----------------------------------------------------------------------------|
| [ROC | $[T]_0$ Economic ₁ nev | ws_2 had $_3$ little | e ₄ effect ₅ | on ₆ financial ₇ markets ₈ . ₉ |
| | | σ | β | A |
| 1. | (| ([0], | $[1,\ldots,9],$ | Ø) |
| 2. | $SHIFT \implies O$ | [0,1], | $[2, \ldots, 9],$ | Ø) |
| 3. | $Left\text{-}Arc_{NMOD} \implies 0$ | [0], | $[2, \ldots, 9],$ | $A_1 = \{(2, NMOD, 1)\}$ |
| 4. | $SHIFT \implies O$ | [0, 2], | $[3,\ldots,9],$ | A_1 |
| 5. | $Left	ext{-}Arc_{\mathtt{SBJ}} \implies 0$ | ([0], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3, SBJ, 2)\} $ |
| 6. | $SHIFT \implies O$ | [0,3], | $[4,\ldots,9],$ | A_2 |
| 7. | $SHIFT \implies O$ | [0, 3, 4], | $[5,\ldots,9],$ | A_2 |
| 8. | $Left-Arc_{NMOD} \implies 0$ | ([0,3], | $[5,\ldots,9],$ | $A_3 = A_2 \cup \{(5, \texttt{NMOD}, 4)\})$ |
| 9. | $SHIFT \implies 0$ | ([0,3,5], | $[6,\ldots,9],$ | A_3 |
| 10. | $SHIFT \implies O$ | [0, 3, 5, 6], | [7, 8, 9], | A_3 |
| 11. | $SHIFT \implies O$ | [0, 3, 5, 6, 7], | [8, 9], | A_3 |
| 12. | $Left\text{-}Arc_{NMOD} \implies ($ | [0, 3, 5, 6], | [8, 9], | $A_4 = A_3 \cup \{(8, \texttt{NMOD}, 7)\})$ |

-PMOD-

| | VMOD— | ✓ SBJ ✓ ✓ | -NMOD-\ | / NMOD— | |
|------|----------------------------------------------|------------------------|------------------------------------|----------------------------------------------------|----|
| [ROC | $[OT]_0$ Economic ₁ ne | ews_2 had_3 little | e ₄ effect ₅ | on_6 financial ₇ markets ₈ | •9 |
| | | σ | β | A | |
| 1. | | ([0], | $[1,\ldots,9],$ | Ø |) |
| 2. | $SHIFT \implies$ | ([0,1], | $[2, \ldots, 9],$ | Ø |) |
| 3. | $Left	ext{-}Arc_{\mathtt{NMOD}} \implies$ | ([0], | $[2, \ldots, 9],$ | $A_1 = \{(2, \texttt{NMOD}, 1)\}$ |) |
| 4. | $SHIFT \implies$ | ([0,2], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left	ext{-}Arc_{\mathtt{SBJ}} \implies$ | ([0], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3, \mathtt{SBJ}, 2)\}$ |) |
| 6. | $SHIFT \implies$ | ([0,3], | $[4, \ldots, 9],$ | A_2 |) |
| 7. | $SHIFT \implies$ | ([0,3,4], | $[5,\ldots,9],$ | A_2 |) |
| 8. | $Left	ext{-}Arc_{NMOD} \implies$ | ([0,3], | $[5,\ldots,9],$ | $A_3=A_2\cup\{(5,\mathtt{NMOD},4)\}$ |) |
| 9. | $SHIFT \implies$ | ([0,3,5], | $[6,\ldots,9],$ | A_3 |) |
| 10. | $SHIFT \implies$ | ([0,3,5,6], | [7, 8, 9], | A_3 |) |
| 11. | $SHIFT \implies$ | ([0,3,5,6,7], | [8, 9], | A_3 |) |
| 12. | $Left	ext{-}Arc_{NMOD} \implies$ | ([0,3,5,6], | [8, 9], | $A_4 = A_3 \cup \{(8, \texttt{NMOD}, 7)\}$ |) |
| 13. | $Right	ext{-}Arc^s_{\mathtt{PMOD}} \implies$ | ([0,3,5], | [6, 9], | $A_5=A_4\cup\{(6,\mathtt{PMOD},8)\}$ |) |

_NMOD__ NMOD__ / -NMOD-____ ∠SBJ-_ [ROOT]₀ Economic₁ news₂ had₃ little₄ effect₅ on₆ financial₇ markets₈ .9 β A σ [0],1. $[1, \ldots, 9],$ Ø 2. [0, 1], $[2, \ldots, 9],$ Shift \Longrightarrow Ø 3. $\mathsf{Left} ext{-}\mathsf{Arc}_{\mathsf{NMOD}} \implies$ [0], $[2, \ldots, 9],$ $A_1 = \{(2, NMOD, 1)\}$ [0, 2],4. Shift \Longrightarrow $[3, \ldots, 9],$ A_1 $A_2 = A_1 \cup \{(3, SBJ, 2)\}$ [0], $[3, \ldots, 9],$ 5. $\mathsf{Left} ext{-}\mathsf{Arc}_{\mathtt{SBJ}} \implies$ 6. [0, 3], $[4, \ldots, 9],$ Shift \Longrightarrow A_2 [0, 3, 4], $[5, \ldots, 9],$ 7. Shift \Longrightarrow $A_3 = A_2 \cup \{(5, NMOD, 4)\}$ 8. [0, 3], $[5, \ldots, 9],$ $Left-Arc_{NMOD} \implies$ 9. [0, 3, 5], $[6, \ldots, 9],$ Shift \Longrightarrow A_3 10. Shift \Longrightarrow [0, 3, 5, 6],[7, 8, 9], A_3 11. [0, 3, 5, 6, 7],[8, 9],Shift \Longrightarrow A_3 12. $Left-Arc_{NMOD} \implies$ [0, 3, 5, 6],[8, 9], $A_4 = A_3 \cup \{(8, NMOD, 7)\}$ 13. $\mathsf{Right} ext{-}\mathsf{Arc}^s_{\mathtt{PMOD}} \implies$ $A_5 = A_4 \cup \{(6, PMOD, 8)\}$ [0, 3, 5],[6, 9], $A_6 = A_5 \cup \{(5, NMOD, 6)\}$ 14. $Right-Arc_{nmod}^s \Longrightarrow$ [0, 3],[5, 9],

 $A_7 = A_6 \cup \{(3, OBJ, 5)\}$

Arc-standard stack-based algorithm

15.

 $Right-Arc^s_{OBJ} \Longrightarrow$

Arc-eager stack-based algorithm

Illustration Arc-standard transition sequence for English example sentence:

[ROOT]₀ Economic₁ news₂ had₃ little₄ effect₅ on₆ financial₇ markets₈ .9 β A σ [0],1. $[1, \ldots, 9],$ Ø 2. [0, 1], $[2, \ldots, 9],$ Shift \Longrightarrow Ø 3. $\mathsf{Left} ext{-}\mathsf{Arc}_{\mathsf{NMOD}} \implies$ [0], $[2, \ldots, 9],$ $A_1 = \{(2, NMOD, 1)\}$ [0, 2],4. Shift \Longrightarrow $[3, \ldots, 9],$ A_1 $A_2 = A_1 \cup \{(3, SBJ, 2)\}$ [0], $[3, \ldots, 9],$ 5. $\mathsf{Left}\text{-}\mathsf{Arc}_{\mathsf{SR},\mathsf{I}} \implies$ 6. [0, 3], $[4, \ldots, 9],$ Shift \Longrightarrow A_2 Shift \Longrightarrow [0, 3, 4], $[5, \ldots, 9],$ 7. $A_3 = A_2 \cup \{(5, NMOD, 4)\}$ 8. [0, 3], $[5, \ldots, 9],$ $Left-Arc_{NMOD} \implies$ 9. [0, 3, 5], $[6, \ldots, 9],$ Shift \Longrightarrow A_3 10. Shift \Longrightarrow [0, 3, 5, 6],[7, 8, 9], A_3 11. [0, 3, 5, 6, 7],[8, 9],Shift \Longrightarrow A_3 12. $\mathsf{Left} ext{-}\mathsf{Arc}_{\mathsf{NMOD}} \implies$ [0, 3, 5, 6],[8, 9], $A_4 = A_3 \cup \{(8, NMOD, 7)\}$ $\mathsf{Right} ext{-}\mathsf{Arc}^s_{\mathtt{PMOD}} \implies$ $A_5 = A_4 \cup \{(6, PMOD, 8)\}$ 13. [0, 3, 5],[6, 9], $Right-Arc^s_{nmod} \implies$ $A_6 = A_5 \cup \{(5, NMOD, 6)\}$ 14. [0, 3],[5, 9],

[3, 9],

[0],

| | | | σ | β | A | |
|-----|-------------------------------------------------|---|------------------|-----------------|--------------------------------------------|---|
| 1. | | (| [0], | $[1,\ldots,9],$ | Ø |) |
| 2. | $SHIFT \implies$ | (| [0,1], | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $Shift \implies$ | (| [0, 2], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left	ext{-}Arc_{\mathtt{SBJ}} \implies$ | (| [0], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3, \mathtt{SBJ}, 2)\}$ |) |
| 6. | $Shift \implies$ | (| [0, 3], | $[4,\ldots,9],$ | A_2 |) |
| 7. | $Shift \implies$ | (| [0, 3, 4], | $[5,\ldots,9],$ | A_2 |) |
| 8. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0, 3], | $[5,\ldots,9],$ | $A_3=A_2\cup\{(5,\mathtt{NMOD},4)\}$ |) |
| 9. | $Shift \implies$ | (| [0, 3, 5], | $[6,\ldots,9],$ | A_3 |) |
| 10. | $Shift \implies$ | (| [0, 3, 5, 6], | [7, 8, 9], | A_3 |) |
| 11. | $Shift \implies$ | (| [0, 3, 5, 6, 7], | [8, 9], | A_3 |) |
| 12. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0, 3, 5, 6], | [8, 9], | $A_4 = A_3 \cup \{(8, \texttt{NMOD}, 7)\}$ |) |
| 13. | $Right	ext{-}Arc^s_{\mathtt{PMOD}} \implies$ | (| [0, 3, 5], | [6, 9], | $A_5 = A_4 \cup \{(6, \texttt{PMOD}, 8)\}$ |) |
| 14. | $Right	ext{-}Arc^s_{\mathtt{NMOD}} \implies$ | (| [0, 3], | [5, 9], | $A_6 = A_5 \cup \{(5, \mathtt{NMOD}, 6)\}$ |) |
| 15. | Right-Arc $_{\mathtt{OBJ}}^{s} \Longrightarrow$ | (| [0], | [3, 9], | $A_7 = A_6 \cup \{(3, \mathtt{OBJ}, 5)\}$ |) |
| 16. | $SHIFT \implies$ | (| [0, 3], | [9], | A_7 |) |

16.

17.

Illustration Arc-standard transition sequence for English example sentence:

```
[ROOT]<sub>0</sub> Economic<sub>1</sub> news<sub>2</sub> had<sub>3</sub> little<sub>4</sub> effect<sub>5</sub> on<sub>6</sub> financial<sub>7</sub> markets<sub>8</sub> .<sub>9</sub>
                                                                               β
                                                                                                    A
                                                      \sigma
                                                      [0],
   1.
                                                                               [1, \ldots, 9],
                                                                                                    Ø
   2.
                                                      [0, 1],
                                                                               [2, \ldots, 9],
                         Shift \Longrightarrow
                                                                                                    Ø
   3.
             \mathsf{Left}	ext{-}\mathsf{Arc}_{\mathsf{NMOD}} \implies
                                                      [0],
                                                                               [2, \ldots, 9],
                                                                                                    A_1 = \{(2, NMOD, 1)\}
                                                      [0, 2],
   4.
                         Shift \Longrightarrow
                                                                               [3, \ldots, 9],
                                                                                                    A_1
                                                                                                    A_2 = A_1 \cup \{(3, SBJ, 2)\}
                                                      [0],
                                                                               [3, \ldots, 9],
   5.
              \mathsf{Left}\text{-}\mathsf{Arc}_{\mathsf{SR},\mathsf{I}} \implies
   6.
                                                      [0, 3],
                                                                               [4, \ldots, 9],
                          Shift \Longrightarrow
                                                                                                    A_2
                                                      [0, 3, 4],
                                                                               [5, \ldots, 9],
   7.
                          Shift \Longrightarrow
                                                                                                    A_3 = A_2 \cup \{(5, NMOD, 4)\}
   8.
                                                      [0, 3],
                                                                               [5, \ldots, 9],
             Left-Arc_{NMOD} \implies
   9.
                                                      [0, 3, 5],
                                                                               [6, \ldots, 9],
                          Shift \Longrightarrow
                                                                                                    A_3
  10.
                          Shift \Longrightarrow
                                                      [0, 3, 5, 6],
                                                                               [7, 8, 9],
                                                                                                    A_3
 11.
                                                                               [8, 9],
                          Shift \Longrightarrow
                                                      [0, 3, 5, 6, 7],
                                                                                                    A_3
 12.
             \mathsf{Left}	ext{-}\mathsf{Arc}_{\mathsf{NMOD}} \implies
                                                      [0, 3, 5, 6],
                                                                               [8, 9],
                                                                                                    A_4 = A_3 \cup \{(8, NMOD, 7)\}
           \mathsf{Right}	ext{-}\mathsf{Arc}^s_{\mathtt{PMOD}} \implies
                                                                                                    A_5 = A_4 \cup \{(6, PMOD, 8)\}
 13.
                                                      [0, 3, 5],
                                                                               [6, 9],
 14.
           Right-Arc_{nmod}^s \Longrightarrow
                                                      [0, 3],
                                                                               [5, 9],
                                                                                                    A_6 = A_5 \cup \{(5, NMOD, 6)\}
 15.
             Right-Arc^s_{OBJ} \Longrightarrow
                                                      [0],
                                                                               [3, 9],
                                                                                                    A_7 = A_6 \cup \{(3, OBJ, 5)\}
```

[9],

[3],

 $A_8 = A_7 \cup \{(3, P, 9)\}$

[0, 3],

[0],

Shift \Longrightarrow $\mathsf{Right}\text{-}\mathsf{Arc}^s_{\mathsf{p}} \implies$

```
[0],
                                                                             [1, \ldots, 9],
  1.
                                                                                                 Ø
  2.
                                                    [0, 1],
                                                                             [2, \ldots, 9],
                        Shift \Longrightarrow
                                                                                                 Ø
  3.
            \mathsf{Left}	ext{-}\mathsf{Arc}_{\mathsf{NMOD}} \implies
                                                    [0],
                                                                             [2, \ldots, 9],
                                                                                                 A_1 = \{(2, NMOD, 1)\}
                                                    [0, 2],
  4.
                        Shift \Longrightarrow
                                                                             [3, \ldots, 9],
                                                                                                 A_1
                                                                                                 A_2 = A_1 \cup \{(3, SBJ, 2)\}
                                                    [0],
  5.
             Left-Arc_{SRJ} \Longrightarrow
                                                                             [3, \ldots, 9],
  6.
                                                    [0, 3].
                                                                             [4, \ldots, 9],
                        Shift \Longrightarrow
                                                                                                 A_2
                                                    [0, 3, 4],
                                                                             [5, \ldots, 9],
  7.
                        Shift \Longrightarrow
                                                                                                 A_3 = A_2 \cup \{(5, \texttt{NMOD}, 4)\}
  8.
                                                    [0, 3],
                                                                             [5, \ldots, 9],
            \mathsf{Left}	ext{-}\mathsf{Arc}_{\mathsf{NMOD}} \implies
 9.
                                                    [0, 3, 5],
                                                                             [6, \ldots, 9],
                        Shift \Longrightarrow
                                                                                                 A_3
10.
                        Shift \Longrightarrow
                                                    [0, 3, 5, 6],
                                                                             [7, 8, 9],
                                                                                                 A_3
                                                                             [8, 9],
11.
                        Shift \Longrightarrow
                                                    [0, 3, 5, 6, 7],
                                                                                                 A_3
12.
            \mathsf{Left}	ext{-}\mathsf{Arc}_{\mathsf{NMOD}} \implies
                                                    [0, 3, 5, 6],
                                                                             [8, 9],
                                                                                                 A_4 = A_3 \cup \{(8, NMOD, 7)\}
         \mathsf{Right}	ext{-}\mathsf{Arc}^s_{\mathtt{PMOD}} \implies
                                                                                                 A_5 = A_4 \cup \{(6, PMOD, 8)\}
13.
                                                    [0, 3, 5],
                                                                             [6, 9],
14.
         Right-Arc_{nmod}^s \Longrightarrow
                                                    [0, 3],
                                                                             [5, 9],
                                                                                                 A_6 = A_5 \cup \{(5, NMOD, 6)\}
15.
           Right-Arc^s_{OBJ} \Longrightarrow
                                                    [0],
                                                                             [3, 9],
                                                                                                 A_7 = A_6 \cup \{(3, OBJ, 5)\}
                                                    [0, 3],
                                                                             [9],
16.
                        Shift \Longrightarrow
                                                                                                 A_7
17.
              Right-Arc_{p}^{s} \Longrightarrow
                                                                             [3],
                                                                                                 A_8 = A_7 \cup \{(3, P, 9)\}
                                                    [0],
18.
         Right-Arc_{rot}^s \Longrightarrow
                                                    Π,
                                                                             [0],
                                                                                                 A_9 = A_8 \cup \{(0, ROOT, 3)\}
```

| | | | | σ | β | A | |
|-----|-------------------------------------|-------------------|---|------------------|-----------------|--------------------------------------------|---|
| 1. | | | (| [0], | $[1,\ldots,9],$ | Ø |) |
| 2. | Shift | \Longrightarrow | (| [0, 1], | $[2,\ldots,9],$ | Ø |) |
| 3. | Left-Arc _{nmod} | \Longrightarrow | (| [0], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | Shift | \Longrightarrow | (| [0, 2], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left	ext{-}Arc_{SBJ}$ | \Longrightarrow | (| [0], | $[3,\ldots,9],$ | $A_2=A_1\cup\{(3,\mathtt{SBJ},2)\}$ |) |
| 6. | Shift | \Longrightarrow | (| [0, 3], | $[4,\ldots,9],$ | A_2 |) |
| 7. | Shift | \Longrightarrow | (| [0, 3, 4], | $[5,\ldots,9],$ | A_2 |) |
| 8. | Left-Arc _{nmod} | \Longrightarrow | (| [0, 3], | $[5,\ldots,9],$ | $A_3=A_2\cup\{(5,\mathtt{NMOD},4)\}$ |) |
| 9. | Shift | \Longrightarrow | (| [0, 3, 5], | $[6,\ldots,9],$ | A_3 |) |
| 10. | Shift | \Longrightarrow | (| [0, 3, 5, 6], | [7, 8, 9], | A_3 |) |
| 11. | Shift | \Longrightarrow | (| [0, 3, 5, 6, 7], | [8, 9], | A_3 |) |
| 12. | Left-Arc _{nmod} | \Longrightarrow | (| [0, 3, 5, 6], | [8, 9], | $A_4 = A_3 \cup \{(8, \texttt{NMOD}, 7)\}$ |) |
| 13. | $Right	ext{-}Arc^s_{\mathtt{PMOD}}$ | \Longrightarrow | (| [0, 3, 5], | [6, 9], | $A_5=A_4\cup\{(6,\mathtt{PMOD},8)\}$ |) |
| 14. | $Right	ext{-}Arc^s_{\mathtt{NMOD}}$ | \Longrightarrow | (| [0, 3], | [5, 9], | $A_6 = A_5 \cup \{(5, \mathtt{NMOD}, 6)\}$ |) |
| 15. | Rіght-Arc $_{\mathtt{OBJ}}^{s}$ | \Longrightarrow | (| [0], | [3, 9], | $A_7 = A_6 \cup \{(3, \mathtt{OBJ}, 5)\}$ |) |
| 16. | Shift | \Longrightarrow | (| [0, 3], | [9], | A_7 |) |
| 17. | $Right	ext{-}Arc^s_{\mathtt{P}}$ | | (| [0], | [3], | $A_8 = A_7 \cup \{(3, P, 9)\}$ |) |
| 18. | Right-Arc $_{\mathtt{ROOT}}^s$ | \Longrightarrow | (| [], | [0], | $A_9 = A_8 \cup \{(0, \mathtt{ROOT}, 3)\}$ |) |
| 19. | Shift | \Longrightarrow | (| [0], | [], | A_9 |) |

Arc-eager stack-based algorithm

| | | | | σ | β | A | |
|-----|-------------------------------------|-------------------|---|------------------|-----------------|--------------------------------------------|---|
| 1. | | | (| [0], | $[1,\ldots,9],$ | Ø |) |
| 2. | Shift | \Longrightarrow | (| [0, 1], | $[2,\ldots,9],$ | Ø |) |
| 3. | Left-Arc _{nmod} | \Longrightarrow | (| [0], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | Shift | \Longrightarrow | (| [0, 2], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left	ext{-}Arc_{SBJ}$ | \Longrightarrow | (| [0], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3,\mathtt{SBJ},2)\}$ |) |
| 6. | Shift | \Longrightarrow | (| [0, 3], | $[4,\ldots,9],$ | A_2 |) |
| 7. | Shift | \Longrightarrow | (| [0, 3, 4], | $[5,\ldots,9],$ | A_2 |) |
| 8. | Left-Arc _{nmod} | \Longrightarrow | (| [0, 3], | $[5,\ldots,9],$ | $A_3=A_2\cup\{(5,\mathtt{NMOD},4)\}$ |) |
| 9. | Shift | \Longrightarrow | (| [0, 3, 5], | $[6,\ldots,9],$ | A_3 |) |
| 10. | Shift | \Longrightarrow | (| [0, 3, 5, 6], | [7, 8, 9], | A_3 |) |
| 11. | Shift | \Longrightarrow | (| [0, 3, 5, 6, 7], | [8, 9], | A_3 |) |
| 12. | Left-Arc _{nmod} | \Longrightarrow | (| [0, 3, 5, 6], | [8, 9], | $A_4 = A_3 \cup \{(8, \texttt{NMOD}, 7)\}$ |) |
| 13. | RIGHT-ARC $_{\mathtt{PMOD}}^{s}$ | \Longrightarrow | (| [0, 3, 5], | [6, 9], | $A_5 = A_4 \cup \{(6, \texttt{PMOD}, 8)\}$ |) |
| 14. | $Right	ext{-}Arc^s_{\mathtt{NMOD}}$ | \Longrightarrow | (| [0, 3], | [5, 9], | $A_6 = A_5 \cup \{(5, \texttt{NMOD}, 6)\}$ |) |
| 15. | $Right	ext{-}Arc^s_{\mathtt{OBJ}}$ | \Longrightarrow | (| [0], | [3, 9], | $A_7 = A_6 \cup \{(3, \mathtt{OBJ}, 5)\}$ |) |
| 16. | Shift | \Longrightarrow | (| [0, 3], | [9], | A_7 |) |
| 17. | $Right	ext{-}Arc^s_{\mathtt{P}}$ | \Longrightarrow | (| [0], | [3], | $A_8 = A_7 \cup \{(3, P, 9)\}$ |) |
| 18. | RIGHT-ARC $_{\mathtt{ROOT}}^{s}$ | \Longrightarrow | (| [], | [0], | $A_9 = A_8 \cup \{(0, \mathtt{ROOT}, 3)\}$ |) |
| 19. | Shift | \Longrightarrow | (| [0], | [], | A_9 |) |

Arc-eager stack-based algorithm

Arc-eager stack-based algorithm

Transitions LEFT-ARC1 $(\sigma|i,j|\beta,A) \Rightarrow (\sigma,j|\beta,A\cup\{(j,l,i)\})$ RIGHT-ARC1 $(\sigma|i,j|\beta,A) \Rightarrow (\sigma|i|j,\beta,A \cup \{(i,l,j)\})$ REDUCE $(\sigma|i,\beta,A) \Rightarrow (\sigma,\beta,A)$ $(\sigma, i | \beta, A) \Rightarrow (\sigma | i, \beta, A)$ SHIFT **Preconditions** LEFT-ARC1 $\neg[i=0]$ $\neg \exists k \exists l' [(k, l', i) \in A]$ RIGHT-ARC^e $\neg \exists k \exists l'[(k,l',j) \in A]$ REDUCE $\exists k \exists l [(k,l,i) \in A]$

Figure: Transitions for the arc-eager stack-based parsing algorithm.

"

The arc-eager parser differs from the arc-standard one by attaching right dependents (using Right-Arc $_l^e$ transitions) as soon as possible, that is, before the right dependent has found all its right dependents.

As a consequence, the Right-Arc $_l^e$ transitions cannot replace the head-dependent structure with the head, as in the arc-standard system, but must store both the head and the dependent on the stack for further processing. The dependent can be popped from the stack at a later time through the Reduce transition, which completes the reduction of this structure.

Arc-eager stack-based algorithm

Illustration Arc-eager transition sequence for English example sentence:

| | σ | β A | |
|----|----------|----------------------------|---|
| 1. | ([0], | $[1,\ldots,9],\varnothing$ |) |

Arc-eager stack-based algorithm

Illustration Arc-eager transition sequence for English example sentence:

| | | σ | β | A |
|----|---------------------------|----------------|-----------------------------------|------------|
| 1. | $SHIFT \Longrightarrow ($ | [0], [0, 1] | $[1, \dots, 9],$ $[2, \dots, 9],$ | Ø) Ø) |

Arc-eager stack-based algorithm

Illustration Arc-eager transition sequence for English example sentence:

| | | | σ | β | A | |
|----|----------------------------------|---|---------|-------------------|-----------------------------------|---|
| 1. | | (| [0], | $[1,\ldots,9],$ | Ø |) |
| 2. | $SHIFT \implies$ | (| [0, 1], | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0], | $[2, \ldots, 9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |

Arc-eager stack-based algorithm

Illustration Arc-eager transition sequence for English example sentence:

| | | | σ | β | A | |
|----|-------------------------------------------|---|---------|-----------------|-----------------------------------|---|
| 1. | | (| [0], | $[1,\ldots,9],$ | Ø |) |
| 2. | $SHIFT \implies$ | (| [0, 1], | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left	ext{-}Arc_{\mathtt{NMOD}} \implies$ | (| [0], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $SHIFT \implies$ | (| [0, 2], | $[3,\ldots,9],$ | A_1 |) |

| | | | σ | β | A | |
|----|-------------------------------------------|---|----------|-----------------|-------------------------------------|---|
| 1. | | (| [0], | $[1,\ldots,9],$ | Ø |) |
| 2. | $SHIFT \implies$ | (| [0, 1], | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left	ext{-}Arc_{\mathtt{NMOD}} \implies$ | (| [0], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $SHIFT \implies$ | (| [0, 2], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left\text{-}Arc_{\mathtt{SBJ}} \implies$ | (| [0], | $[3,\ldots,9],$ | $A_2=A_1\cup\{(3,\mathtt{SBJ},2)\}$ |) |

Arc-eager stack-based algorithm

Illustration Arc-eager transition sequence for English example sentence:

| | | | σ | eta | A | |
|----|----------------------------------------------|---|----------|-------------------|--------------------------------------------|---|
| 1. | | (| [0], | $[1,\ldots,9],$ | Ø |) |
| 2. | $SHIFT \implies$ | (| [0, 1], | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left	ext{-}Arc_{\mathtt{NMOD}} \implies$ | (| [0], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $SHIFT \implies$ | (| [0, 2], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left\text{-}Arc_{\mathtt{SBJ}} \implies$ | (| [0], | $[3,\ldots,9],$ | $A_2=A_1\cup\{(3,\mathtt{SBJ},2)\}$ |) |
| 6. | $Right	ext{-}Arc^e_{\mathtt{ROOT}} \implies$ | (| [0, 3], | $[4, \ldots, 9],$ | $A_3 = A_2 \cup \{(0, \mathtt{ROOT}, 3)\}$ |) |

Arc-eager stack-based algorithm

Illustration Arc-eager transition sequence for English example sentence:

 $[\mathsf{ROOT}]_0 \ \mathsf{Economic}_1 \ \mathsf{news}_2 \ \mathsf{had}_3 \ \mathsf{little}_4 \ \mathsf{effect}_5 \ \mathsf{on}_6 \ \mathsf{financial}_7 \ \mathsf{markets}_8 \ .9$

| | | | σ | β | A | |
|----|----------------------------------------------|---|------------|-------------------|--------------------------------------|---|
| 1. | | (| [0], | $[1,\ldots,9],$ | Ø |) |
| 2. | $SHIFT \implies$ | (| [0, 1], | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left	ext{-}Arc_{\mathtt{NMOD}} \implies$ | (| [0], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $SHIFT \implies$ | (| [0, 2], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left\text{-}Arc_{\mathtt{SBJ}} \implies$ | (| [0], | $[3,\ldots,9],$ | $A_2=A_1\cup\{(3,\mathtt{SBJ},2)\}$ |) |
| 6. | $Right	ext{-}Arc^e_{\mathtt{ROOT}} \implies$ | (| [0, 3], | $[4,\ldots,9],$ | $A_3=A_2\cup\{(0,\mathtt{ROOT},3)\}$ |) |
| 7. | $SHIFT \implies$ | (| [0, 3, 4], | $[5, \ldots, 9],$ | A_3 |) |

Arc-eager stack-based algorithm

Illustration Arc-eager transition sequence for English example sentence:

| | | | σ | β | | A | |
|----|----------------------------------------------|---|------------|---------|--------------|--------------------------------------------|---|
| 1. | | (| [0], | [1, . | ,9], | Ø |) |
| 2. | $SHIFT \implies$ | (| [0, 1], | [2, . | $\dots, 9],$ | Ø |) |
| 3. | $Left	ext{-}Arc_{\mathtt{NMOD}} \implies$ | (| [0], | [2, . | $\dots, 9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $SHIFT \implies$ | (| [0, 2], | [3, . | $\dots, 9],$ | A_1 |) |
| 5. | $Left\text{-}Arc_{\mathtt{SBJ}} \implies$ | (| [0], | [3, . | $\dots, 9],$ | $A_2 = A_1 \cup \{(3, \mathtt{SBJ}, 2)\}$ |) |
| 6. | $Right	ext{-}Arc^e_{\mathtt{ROOT}} \implies$ | (| [0, 3], | [4, . | $\dots, 9],$ | $A_3 = A_2 \cup \{(0, \mathtt{ROOT}, 3)\}$ |) |
| 7. | $SHIFT \implies$ | (| [0, 3, 4], | [5, . | $\dots, 9],$ | A_3 |) |
| 8. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0, 3], | [5,] | , 9], | $A_4 = A_3 \cup \{(5, NMOD, 4)\}$ |) |

Arc-eager stack-based algorithm

| | | | σ | β | A |
|----|----------------------------------------------|---|------------|-----------------|-----------------------------------------------|
| 1. | | (| [0], | $[1,\ldots,9],$ | Ø) |
| 2. | $SHIFT \implies$ | (| [0, 1], | $[2,\ldots,9],$ | Ø) |
| 3. | $Left	ext{-}Arc_{\mathtt{NMOD}} \implies$ | (| [0], | $[2,\ldots,9],$ | $A_1 = \{(2, \texttt{NMOD}, 1)\}$ |
| 4. | $SHIFT \implies$ | (| [0, 2], | $[3,\ldots,9],$ | A_1) |
| 5. | $Left\text{-}Arc_{\mathtt{SBJ}} \implies$ | (| [0], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3, SBJ, 2)\}$ |
| 6. | $Right	ext{-}Arc^e_{\mathtt{ROOT}} \implies$ | (| [0, 3], | $[4,\ldots,9],$ | $A_3 = A_2 \cup \{(0, \mathtt{ROOT}, 3)\}$) |
| 7. | $SHIFT \implies$ | (| [0, 3, 4], | $[5,\ldots,9],$ | A_3 |
| 8. | $Left	ext{-}Arc_{\mathtt{NMOD}} \implies$ | (| [0, 3], | $[5,\ldots,9],$ | $A_4 = A_3 \cup \{(5, \texttt{NMOD}, 4)\})$ |
| 9. | Rіght-Arc $_{\mathtt{OBJ}}^{e} \implies$ | (| [0, 3, 5], | $[6,\ldots,9],$ | $A_5 = A_4 \cup \{(3, OBJ, 5)\}$ |

Arc-eager stack-based algorithm

```
NMOD— SBJ—OBJ—NMOD—NMOD—
[ROOT]<sub>0</sub> Economic<sub>1</sub> news<sub>2</sub> had<sub>3</sub> little<sub>4</sub> effect<sub>5</sub> on<sub>6</sub> financial<sub>7</sub> markets<sub>8</sub> .9
```

| | | | σ | β | A | |
|-----|-------------------------------------------------|---|---------------|-------------------------|--------------------------------------------|---|
| 1. | | (| [0], | $\boxed{[1,\ldots,9],}$ | Ø |) |
| 2. | $SHIFT \implies$ | (| [0, 1], | $[2, \ldots, 9],$ | Ø |) |
| 3. | $Left	ext{-}Arc_{\mathtt{NMOD}} \implies$ | (| [0], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $SHIFT \implies$ | (| [0, 2], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left	ext{-}Arc_{\mathtt{SBJ}} \implies$ | (| [0], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3, \mathtt{SBJ}, 2)\}$ |) |
| 6. | $Right	ext{-}Arc^e_{\mathtt{ROOT}} \implies$ | (| [0, 3], | $[4,\ldots,9],$ | $A_3=A_2\cup\{(0,\mathtt{ROOT},3)\}$ |) |
| 7. | $SHIFT \implies$ | (| [0, 3, 4], | $[5,\ldots,9],$ | A_3 |) |
| 8. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0, 3], | $[5,\ldots,9],$ | $A_4 = A_3 \cup \{(5, \mathtt{NMOD}, 4)\}$ |) |
| 9. | Rіgнт-Arc $_{\mathtt{OBJ}}^{e} \Longrightarrow$ | (| [0, 3, 5], | $[6,\ldots,9],$ | $A_5 = A_4 \cup \{(3, \mathtt{OBJ}, 5)\}$ |) |
| 10. | $Right	ext{-}Arc^e_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [7, 8, 9], | $A_6 = A_5 \cup \{(5, \mathtt{NMOD}, 6)\}$ |) |

| | | | σ | β | A | |
|-----|----------------------------------------------|---|------------------|-----------------|--------------------------------------------|---|
| 1. | | (| [0], | $[1,\ldots,9],$ | Ø |) |
| 2. | $SHIFT \implies$ | (| [0, 1], | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $SHIFT \implies$ | (| [0, 2], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left\text{-}Arc_{\mathtt{SBJ}} \implies$ | (| [0], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3,\mathtt{SBJ},2)\}$ |) |
| 6. | $Right	ext{-}Arc^e_{\mathtt{ROOT}} \implies$ | (| [0, 3], | $[4,\ldots,9],$ | $A_3=A_2\cup\{(0,\mathtt{ROOT},3)\}$ |) |
| 7. | $SHIFT \implies$ | (| [0, 3, 4], | $[5,\ldots,9],$ | A_3 |) |
| 8. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0, 3], | $[5,\ldots,9],$ | $A_4 = A_3 \cup \{(5, \mathtt{NMOD}, 4)\}$ |) |
| 9. | $Right	ext{-}Arc^e_{\mathtt{OBJ}} \implies$ | (| [0, 3, 5], | $[6,\ldots,9],$ | $A_5 = A_4 \cup \{(3, \mathtt{OBJ}, 5)\}$ |) |
| 10. | $Right	ext{-}Arc^e_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [7, 8, 9], | $A_6 = A_5 \cup \{(5, \mathtt{NMOD}, 6)\}$ |) |
| 11. | $SHIFT \implies$ | (| [0, 3, 5, 6, 7], | [8, 9], | A_6 |) |

Arc-eager stack-based algorithm

Illustration Arc-eager transition sequence for English example sentence:

| | | | σ | β | A | |
|-----|-------------------------------------------------|---|------------------|-----------------|--------------------------------------------|---|
| 1. | | (| [0], | $[1,\ldots,9],$ | Ø |) |
| 2. | $SHIFT \implies$ | (| [0,1], | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0], | $[2,\ldots,9],$ | $A_1 = \{(2, \texttt{NMOD}, 1)\}$ |) |
| 4. | $SHIFT \implies$ | (| [0, 2], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left	ext{-}Arc_{\mathtt{SBJ}} \implies$ | (| [0], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3, SBJ, 2)\}$ |) |
| 6. | $Right	ext{-}Arc^e_{\mathtt{ROOT}} \implies$ | (| [0, 3], | $[4,\ldots,9],$ | $A_3 = A_2 \cup \{(0, \mathtt{ROOT}, 3)\}$ |) |
| 7. | $SHIFT \implies$ | (| [0, 3, 4], | $[5,\ldots,9],$ | A_3 |) |
| 8. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0, 3], | $[5,\ldots,9],$ | $A_4 = A_3 \cup \{(5, \mathtt{NMOD}, 4)\}$ |) |
| 9. | Rіght-Arc $_{\mathtt{OBJ}}^{e} \Longrightarrow$ | (| [0, 3, 5], | $[6,\ldots,9],$ | $A_5 = A_4 \cup \{(3, \mathtt{OBJ}, 5)\}$ |) |
| 10. | $Right	ext{-}Arc^e_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [7, 8, 9], | $A_6 = A_5 \cup \{(5, \texttt{NMOD}, 6)\}$ |) |
| 11. | $SHIFT \implies$ | (| [0, 3, 5, 6, 7], | [8, 9], | A_6 |) |
| 12. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0, 3, 5, 6], | [8, 9], | $A_7 = A_6 \cup \{(8, NMOD, 7)\}$ |) |

| | | | σ | β | A | |
|-----|----------------------------------------------|---|------------------|-------------------|--------------------------------------------|---|
| 1. | | (| [0], | $[1,\ldots,9],$ | Ø |) |
| 2. | Shift \Longrightarrow | (| [0, 1], | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0], | $[2, \ldots, 9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $Shift \implies$ | (| [0, 2], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left	ext{-}Arc_{\mathtt{SBJ}} \implies$ | (| [0], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3,\mathtt{SBJ},2)\}$ |) |
| 6. | $Right	ext{-}Arc^e_{\mathtt{ROOT}} \implies$ | (| [0, 3], | $[4,\ldots,9],$ | $A_3 = A_2 \cup \{(0, \mathtt{ROOT}, 3)\}$ |) |
| 7. | $SHIFT \implies$ | (| [0, 3, 4], | $[5,\ldots,9],$ | A_3 |) |
| 8. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0, 3], | $[5,\ldots,9],$ | $A_4 = A_3 \cup \{(5, \mathtt{NMOD}, 4)\}$ |) |
| 9. | $Right	ext{-}Arc^e_{OBJ} \implies$ | (| [0, 3, 5], | $[6, \ldots, 9],$ | $A_5 = A_4 \cup \{(3, \mathtt{OBJ}, 5)\}$ |) |
| 10. | $Right	ext{-}Arc^e_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [7, 8, 9], | $A_6 = A_5 \cup \{(5, \mathtt{NMOD}, 6)\}$ |) |
| 11. | $Shift \implies$ | (| [0, 3, 5, 6, 7], | [8, 9], | A_6 |) |
| 12. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0, 3, 5, 6], | [8, 9], | $A_7 = A_6 \cup \{(8, \mathtt{NMOD}, 7)\}$ |) |
| 13. | $Right	ext{-}Arc^e_{PMOD} \implies$ | (| [0, 3, 5, 6, 8], | [9], | $A_8 = A_7 \cup \{(6, \texttt{PMOD}, 8)\}$ |) |

| | | σ | β | A | |
|-----|------------------------------------------------|------------------|-----------------|--------------------------------------------|---|
| 1. | (| [0], | $[1,\ldots,9],$ | Ø |) |
| 2. | $SHIFT \implies ($ | [0, 1], | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left\text{-}Arc_{\mathtt{NMOD}} \implies ($ | [0], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $SHIFT \implies ($ | [0, 2], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left\text{-}Arc_{\mathtt{SBJ}} \implies ($ | [0], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3, \mathtt{SBJ}, 2)\}$ |) |
| 6. | $Right	ext{-}Arc^e_{\mathtt{ROOT}} \implies ($ | [0, 3], | $[4,\ldots,9],$ | $A_3=A_2\cup\{(0,\mathtt{ROOT},3)\}$ |) |
| 7. | $SHIFT \implies ($ | [0, 3, 4], | $[5,\ldots,9],$ | A_3 |) |
| 8. | $Left\text{-}Arc_{\mathtt{NMOD}} \implies ($ | [0, 3], | $[5,\ldots,9],$ | $A_4 = A_3 \cup \{(5, \mathtt{NMOD}, 4)\}$ |) |
| 9. | $Right	ext{-}Arc^e_{\mathtt{OBJ}} \implies ($ | [0, 3, 5], | $[6,\ldots,9],$ | $A_5 = A_4 \cup \{(3, \mathtt{OBJ}, 5)\}$ |) |
| 10. | $Right	ext{-}Arc^e_{\mathtt{NMOD}} \implies ($ | [0, 3, 5, 6], | [7, 8, 9], | $A_6 = A_5 \cup \{(5, \mathtt{NMOD}, 6)\}$ |) |
| 11. | $SHIFT \implies ($ | [0, 3, 5, 6, 7], | [8, 9], | A_6 |) |
| 12. | $Left\text{-}Arc_{\mathtt{NMOD}} \implies ($ | [0, 3, 5, 6], | [8, 9], | $A_7 = A_6 \cup \{(8, \mathtt{NMOD}, 7)\}$ |) |
| 13. | $Right	ext{-}Arc^e_{\mathtt{PMOD}} \implies ($ | [0, 3, 5, 6, 8], | [9], | $A_8 = A_7 \cup \{(6, \texttt{PMOD}, 8)\}$ |) |
| 14. | Reduce \Longrightarrow (| [0, 3, 5, 6], | [9], | A_8 |) |

2 Stack-based Algorithms

 $\mathsf{Right}\text{-}\mathsf{Arc}^e_{\mathtt{PMOD}} \implies$

Reduce \Longrightarrow

Reduce \Longrightarrow

13.

14.

15.

Illustration Arc-eager transition sequence for English example sentence:

```
OBJ_NMOD_\\\ \rangle NMOD_\\\\ \rangle \rangle
[ROOT]<sub>0</sub> Economic<sub>1</sub> news<sub>2</sub> had<sub>3</sub> little<sub>4</sub> effect<sub>5</sub> on<sub>6</sub> financial<sub>7</sub> markets<sub>8</sub> .9
                                                                                                                                                                                                                                                                                  A
                                                                                                                                                    \sigma
         1.
                                                                                                                                                     [0],
                                                                                                                                                                                                                          [1, \ldots, 9],
                                                                                                                                                                                                                                                                                  Ø
         2.
                                                                                                                                                                                                                          [2, \ldots, 9],
                                                                      Shift \Longrightarrow
                                                                                                                                                     [0, 1],
                                                                                                                                                                                                                                                                                  A_1 = \{(2, NMOD, 1)\}
          3.
                                     \mathsf{Left}	ext{-}\mathsf{Arc}_{\mathsf{NMOD}} \implies
                                                                                                                                                     [0],
                                                                                                                                                                                                                          [2, \ldots, 9],
          4.
                                                                                                                                                     [0, 2],
                                                                                                                                                                                                                          [3, \ldots, 9],
                                                                                                                                                                                                                                                                                  A_1
                                                                       Shift \Longrightarrow
                                                                                                                                                                                                                                                                                 A_2 = A_1 \cup \{(3, SBJ, 2)\}
                                                                                                                                                                                                                          [3, \ldots, 9],
         5.
                                        \mathsf{Left}	ext{-}\mathsf{Arc}_{\mathsf{SBJ}} \implies
                                                                                                                                                     [0],
         6.
                               \mathsf{Right}	ext{-}\mathsf{Arc}^e_{\mathtt{ROOT}} \implies
                                                                                                                                                     [0, 3],
                                                                                                                                                                                                                          [4, \ldots, 9],
                                                                                                                                                                                                                                                                                  A_3 = A_2 \cup \{(0, ROOT, 3)\}
         7.
                                                                                                                                                     [0, 3, 4],
                                                                                                                                                                                                                          [5, \ldots, 9],
                                                                      Shift \Longrightarrow
                                                                                                                                                                                                                          [5,\ldots,9],
         8.
                                                                                                                                                     [0, 3],
                                                                                                                                                                                                                                                                                  A_4 = A_3 \cup \{(5, NMOD, 4)\}
                                    Left-Arc_{NMOD} \implies
                                                                                                                                                                                                                          [6, \ldots, 9],
                                   Right-Arc_{\mathtt{OB,I}}^{e} \Longrightarrow
                                                                                                                                                                                                                                                                                 A_5 = A_4 \cup \{(3, OBJ, 5)\}
         9.
                                                                                                                                                     [0, 3, 5],
                               Right-Arc^e_{nmod} \implies
                                                                                                                                                     [0, 3, 5, 6],
                                                                                                                                                                                                                         [7, 8, 9],
                                                                                                                                                                                                                                                                                 A_6 = A_5 \cup \{(5, NMOD, 6)\}
     10.
   11.
                                                                      Shift \Longrightarrow
                                                                                                                                                    [0, 3, 5, 6, 7],
                                                                                                                                                                                                                         [8, 9],
                                                                                                                                                                                                                                                                                  A_6
   12.
                                     Left-Arc_{NMOD} \implies
                                                                                                                                                    [0, 3, 5, 6],
                                                                                                                                                                                                                          [8, 9],
                                                                                                                                                                                                                                                                                 A_7 = A_6 \cup \{(8, NMOD, 7)\}
```

[9],

[9],

[9],

 $A_8 = A_7 \cup \{(6, PMOD, 8)\}$

 A_8

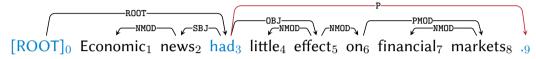
 A_8

[0, 3, 5, 6, 8],

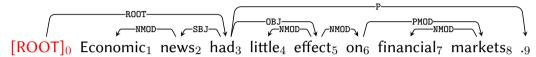
[0, 3, 5, 6],

[0, 3, 5],

| | | | σ | eta | A | |
|-----|----------------------------------------------|---|------------------|-----------------|--------------------------------------------|---|
| 1. | | (| [0], | $[1,\ldots,9],$ | Ø |) |
| 2. | $SHIFT \implies$ | (| [0,1], | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $Shift \implies$ | (| [0, 2], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left\text{-}Arc_{\mathtt{SBJ}} \implies$ | (| [0], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3, \mathtt{SBJ}, 2)\}$ |) |
| 6. | $Right	ext{-}Arc^e_{\mathtt{ROOT}} \implies$ | (| [0, 3], | $[4,\ldots,9],$ | $A_3=A_2\cup\{(0,\mathtt{ROOT},3)\}$ |) |
| 7. | $SHIFT \implies$ | (| [0, 3, 4], | $[5,\ldots,9],$ | A_3 |) |
| 8. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0, 3], | $[5,\ldots,9],$ | $A_4 = A_3 \cup \{(5, \mathtt{NMOD}, 4)\}$ |) |
| 9. | Rіght-Arc $_{	t OBJ}^e \Longrightarrow$ | (| [0, 3, 5], | $[6,\ldots,9],$ | $A_5 = A_4 \cup \{(3, \mathtt{OBJ}, 5)\}$ |) |
| 10. | $Right	ext{-}Arc^e_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [7, 8, 9], | $A_6 = A_5 \cup \{(5, \mathtt{NMOD}, 6)\}$ |) |
| 11. | $Shift \implies$ | (| [0, 3, 5, 6, 7], | [8, 9], | A_6 |) |
| 12. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0, 3, 5, 6], | [8, 9], | $A_7 = A_6 \cup \{(8, \mathtt{NMOD}, 7)\}$ |) |
| 13. | $Right	ext{-}Arc^e_{\mathtt{PMOD}} \implies$ | (| [0, 3, 5, 6, 8], | [9], | $A_8 = A_7 \cup \{(6, \texttt{PMOD}, 8)\}$ |) |
| 14. | Reduce \Longrightarrow | (| [0, 3, 5, 6], | [9], | A_8 |) |
| 15. | Reduce \Longrightarrow | (| [0, 3, 5], | [9], | A_8 |) |
| 16. | Reduce \Longrightarrow | (| [0, 3], | [9], | A_8 |) |



| | | | σ | β | A | |
|-----|----------------------------------------------|---|------------------|-----------------|--------------------------------------------|---|
| 1. | | (| [0], | $[1,\ldots,9],$ | Ø |) |
| 2. | Shift \Longrightarrow | (| [0, 1], | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | Shift \Longrightarrow | (| [0, 2], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left	ext{-}Arc_{\mathtt{SBJ}} \implies$ | (| [0], | $[3,\ldots,9],$ | $A_2=A_1\cup\{(3,\mathtt{SBJ},2)\}$ |) |
| 6. | $Right	ext{-}Arc^e_{\mathtt{ROOT}} \implies$ | (| [0, 3], | $[4,\ldots,9],$ | $A_3=A_2\cup\{(0,\mathtt{ROOT},3)\}$ |) |
| 7. | Shift \Longrightarrow | (| [0, 3, 4], | $[5,\ldots,9],$ | A_3 |) |
| 8. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0, 3], | $[5,\ldots,9],$ | $A_4 = A_3 \cup \{(5, \mathtt{NMOD}, 4)\}$ |) |
| 9. | Rіght-Arc $_{	t OBJ}^e \Longrightarrow$ | (| [0, 3, 5], | $[6,\ldots,9],$ | $A_5 = A_4 \cup \{(3, \mathtt{OBJ}, 5)\}$ |) |
| 10. | $Right	ext{-}Arc^e_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [7, 8, 9], | $A_6=A_5\cup\{(5,\mathtt{NMOD},6)\}$ |) |
| 11. | Shift \Longrightarrow | (| [0, 3, 5, 6, 7], | [8, 9], | A_6 |) |
| 12. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0, 3, 5, 6], | [8, 9], | $A_7 = A_6 \cup \{(8, \mathtt{NMOD}, 7)\}$ |) |
| 13. | $Right	ext{-}Arc^e_{\mathtt{PMOD}} \implies$ | (| [0, 3, 5, 6, 8], | [9], | $A_8 = A_7 \cup \{(6, \texttt{PMOD}, 8)\}$ |) |
| 14. | Reduce \Longrightarrow | (| [0, 3, 5, 6], | [9], | A_8 |) |
| 15. | Reduce \Longrightarrow | (| [0, 3, 5], | [9], | A_8 |) |
| 16. | Reduce \Longrightarrow | (| [0, 3], | [9], | A_8 |) |
| 17. | $Right	ext{-}Arc^e_{\mathtt{P}} \implies$ | (| [0, 3, 9], | [], | $A_9 = A_8 \cup \{(3, P, 9)\}$ |) |



| | | | σ | β | A | |
|-----|-------------------------------------------------|---|------------------|-----------------|--------------------------------------------|---|
| 1. | | (| [0], | $[1,\ldots,9],$ | Ø |) |
| 2. | $SHIFT \implies$ | (| [0,1], | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $SHIFT \implies$ | (| [0, 2], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left\text{-}Arc_{\mathtt{SBJ}} \implies$ | (| [0], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3, \mathtt{SBJ}, 2)\}$ |) |
| 6. | Rіght-Arc $_{	t ROOT}^e \Longrightarrow$ | (| [0, 3], | $[4,\ldots,9],$ | $A_3=A_2\cup\{(0,\mathtt{ROOT},3)\}$ |) |
| 7. | $SHIFT \implies$ | (| [0, 3, 4], | $[5,\ldots,9],$ | A_3 |) |
| 8. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0, 3], | $[5,\ldots,9],$ | $A_4 = A_3 \cup \{(5, \mathtt{NMOD}, 4)\}$ |) |
| 9. | Rіснт-Arc $_{\mathtt{OBJ}}^{e} \Longrightarrow$ | (| [0, 3, 5], | $[6,\ldots,9],$ | $A_5 = A_4 \cup \{(3, \mathtt{OBJ}, 5)\}$ |) |
| 10. | $Right	ext{-}Arc^e_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [7, 8, 9], | $A_6=A_5\cup\{(5,\mathtt{NMOD},6)\}$ |) |
| 11. | $SHIFT \implies$ | (| [0, 3, 5, 6, 7], | [8, 9], | A_6 |) |
| 12. | $Left	ext{-}Arc_{NMOD} \implies$ | (| [0, 3, 5, 6], | [8, 9], | $A_7 = A_6 \cup \{(8, \mathtt{NMOD}, 7)\}$ |) |
| 13. | $Right	ext{-}Arc^e_{\mathtt{PMOD}} \implies$ | (| [0, 3, 5, 6, 8], | [9], | $A_8 = A_7 \cup \{(6, \texttt{PMOD}, 8)\}$ |) |
| 14. | Reduce \Longrightarrow | (| [0, 3, 5, 6], | [9], | A_8 |) |
| 15. | Reduce \Longrightarrow | (| [0, 3, 5], | [9], | A_8 |) |
| 16. | Reduce \Longrightarrow | (| [0, 3], | [9], | A_8 |) |
| 17. | $Right	ext{-}Arc^e_{\mathtt{P}} \implies$ | (| [0, 3, 9], | [], | $A_9 = A_8 \cup \{(3, P, 9)\}$ |) |

List-based Algorithms

Definition

A list-based configuration for a sentence $x = (w_0, w_1, \dots, w_n)$ is a quadruple $c = (\lambda_1, \lambda_2, \beta, A)$, where

- 1. λ_1 is a list of tokens $i_1 \leq k_1$ (for some $k_1 \leq n$)
 - will represent as a list with head to right (nodes in decreasing order)
- 2. λ_2 is a list of tokens $i_2 \leq k_2$ (for some k_2 , such that $k_1 < k_2 \leq n$)
 - will represent as a list with head to left (nodes in increasing order)
- 3. β is a buffer of tokens $i > k_2$,
 - will represent as a list with head to left
- 4. A is a set of dependency arcs such that $G = (\{0, 1, \dots, n\}, A)$ is a dependency graph for x.

Write $\lambda_1.\lambda_2$ for the concatenation of lists λ_1 and λ_2 . Ex., [0,1].[2,3,4] = [0,1,2,3,4].

Definition

A list-based transition system is a quadruple $S = (C, T, c_{\text{start}}, C_{\text{terminal}})$, where

- 1. C is the set of all list-based configurations,
- 2. $c_{\text{start}}(x = (w_0, w_1, \dots, w_n)) = ([0], [], [1, \dots, n], \emptyset),$
- 3. T is a set of transitions, each of which is a function $t: C \to C$,
- 4. $C_{\text{terminal}} = \{c \in C \mid c = (\lambda_1, \lambda_2, [], A)\}.$

(Note, only difference from stack-based system is: two lists instead of a single stack)

Transitions LEFT-ARC $_{1}^{n}$ $(\lambda_1|i,\lambda_2,i|\beta,A) \Rightarrow (\lambda_1,i|\lambda_2,i|\beta,A\cup\{(i,l,i)\})$ $(\lambda_1|i,\lambda_2,i|\beta,A) \Rightarrow (\lambda_1,i|\lambda_2,i|\beta,A\cup\{(i,l,i)\})$ RIGHT-ARC n $No-Arc^n$ $(\lambda_1|i,\lambda_2,\beta,A) \Rightarrow (\lambda_1,i|\lambda_2,\beta,A)$ $SHIFT^{\lambda}$ $(\lambda_1, \lambda_2, i | \beta, A) \Rightarrow (\lambda_1, \lambda_2 | i, [], \beta, A)$ Preconditions LEFT-ARC $_{1}^{n}$ $\neg[i=0]$ $\neg \exists k \exists l' [(k, l', i) \in A]$ $\neg[i \rightarrow^* j]_A$ $RIGHT-ARC_{1}^{n}$ $\neg \exists k \exists l'[(k,l',j) \in A]$ $\neg [i \rightarrow^* i]_A$

Figure: Transitions for the arc-eager stack-based parsing algorithm.

The fact that both the head and the dependent are kept in either λ_2 or β makes it possible to construct non-projective dependency graphs, because the No-Arcⁿ transition allows a node to be passed from λ_1 to λ_2 even if it does not (yet) have a head.

Projective list-based algorithm

Illustration Transition sequence for non-projective Czech example sentence:

 $[\mathsf{ROOT}]_0 \ \mathsf{Z}_1 \ \mathsf{nich}_2 \ \mathsf{je}_3 \ \mathsf{jen}_4 \ \mathsf{jedna}_5 \ \mathsf{na}_6 \ \mathsf{kvalitu}_7 \ ._8$

| | | λ_1 | λ_2 | β | A |
|----|---|-------------|-------------|-----------------|-----|
| 1. | (| [0], | [], | $[1,\ldots,8],$ | Ø) |

Projective list-based algorithm

 $[\mathsf{ROOT}]_0 \ \mathsf{Z}_1 \ \mathsf{nich}_2 \ \mathsf{je}_3 \ \mathsf{jen}_4 \ \mathsf{jedna}_5 \ \mathsf{na}_6 \ \mathsf{kvalitu}_7 \ ._8$

| | | λ_1 | λ_2 | eta | A | |
|----|--------------------------|-------------|-------------|-----------------|---|---|
| 1. | | ([0], | [], | $[1,\ldots,8],$ | Ø |) |
| 2. | $Shift^\lambda \implies$ | ([0,1], | [], | $[2,\ldots,8],$ | Ø |) |

Projective list-based algorithm

$$[\mathsf{ROOT}]_0$$
 Z_1 nich_2 je_3 jen_4 jedna_5 na_6 $\mathsf{kvalitu}_7$.8

| | | | λ_1 | λ_2 | $oldsymbol{eta}$ | A | |
|----|----------------------------------------------------|---|-------------|-------------|-------------------|--------------------------|---|
| 1. | | (| [0], | [], | $[1,\ldots,8],$ | Ø |) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | [] | $[2, \ldots, 8],$ | Ø |) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \Longrightarrow$ | (| [0], | [1], | $[2,\ldots,8],$ | $A_1=(1,\mathtt{Atr},2)$ |) |

Projective list-based algorithm

$$[ROOT]_0$$
 Z_1 $nich_2$ je_3 jen_4 $jedna_5$ na_6 $kvalitu_7$.8

| | | | λ_1 | λ_2 | $oldsymbol{eta}$ | A | |
|----|---------------------------------------------|---|-------------|-------------|------------------|--------------------------|---|
| 1. | | (| [0], | [], | $[1,\ldots,8],$ | Ø |) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | [] | $[2,\ldots,8],$ | Ø |) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \implies$ | (| [0], | [1], | $[2,\ldots,8],$ | $A_1=(1,\mathtt{Atr},2)$ |) |
| 4. | $Shift^\lambda \Longrightarrow$ | (| [0, 1, 2], | [] | $[3,\ldots,8],$ | A_1 |) |

 $[ROOT]_0$ Z_1 nich $_2$ je $_3$ jen $_4$ jedna $_5$ na $_6$ kvalitu $_7$. $_8$

| | | | λ_1 | λ_2 | β | A | |
|----|----------------------------------------------------|---|-------------|-------------|-----------------|--------------------------|---|
| 1. | | (| [0], | [], | $[1,\ldots,8],$ | Ø |) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | [], | $[2,\ldots,8],$ | Ø |) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \Longrightarrow$ | (| [0], | [1], | $[2,\ldots,8],$ | $A_1=(1,\mathtt{Atr},2)$ |) |
| 4. | $Shift^\lambda \implies$ | (| [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1 |) |
| 5. | $No	ext{-}Arc^n \implies$ | (| [0, 1], | [2], | $[3,\ldots,8],$ | A_1 |) |

Illustration Transition sequence for non-projective Czech example sentence:

 $[\mathsf{ROOT}]_0 \ \mathsf{Z}_1 \ \mathsf{nich}_2 \ \mathsf{je}_3 \ \mathsf{jen}_4 \ \mathsf{jedna}_5 \ \mathsf{na}_6 \ \mathsf{kvalitu}_7 \ ._8$

| | | | λ_1 | λ_2 | $oldsymbol{eta}$ | A | |
|----|---------------------------------------------|---|-------------|-------------|------------------|--------------------------|---|
| 1. | | (| [0], | [], | $[1,\ldots,8],$ | Ø |) |
| 2. | $Shift^\lambda \Longrightarrow$ | (| [0, 1], | [] | $[2,\ldots,8],$ | Ø |) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \implies$ | (| [0], | [1], | $[2,\ldots,8],$ | $A_1=(1,\mathtt{Atr},2)$ |) |
| 4. | $Shift^\lambda \Longrightarrow$ | (| [0, 1, 2], | [] | $[3,\ldots,8],$ | A_1 |) |
| 5. | $No	ext{-}Arc^n \implies$ | (| [0, 1], | [2], | $[3,\ldots,8],$ | A_1 |) |
| 6. | $No	ext{-}Arc^n \implies$ | (| [0], | [1, 2], | $[3,\ldots,8],$ | A_1 |) |

| | | | λ_1 | λ_2 | β | A | |
|----|-----------------------------------------------------|---|-------------|-------------|-----------------|----------------------------------------|---|
| 1. | | (| [0], | [], | $[1,\ldots,8],$ | Ø |) |
| 2. | $Shift^\lambda \Longrightarrow$ | (| [0, 1], | [], | $[2,\ldots,8],$ | Ø |) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \implies$ | (| [0], | [1], | $[2,\ldots,8],$ | $A_1=(1,\mathtt{Atr},2)$ |) |
| 4. | $Shift^\lambda \Longrightarrow$ | (| [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1 |) |
| 5. | $No	ext{-}Arc^n \implies$ | (| [0, 1], | [2], | $[3,\ldots,8],$ | A_1 |) |
| 6. | $No\text{-}Arc^n \implies$ | (| [0], | [1, 2], | $[3,\ldots,8],$ | A_1 |) |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \Longrightarrow$ | (| [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \mathtt{Pred}, 3)$ |) |

Projective list-based algorithm

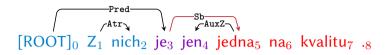
| | | | λ_1 | λ_2 | $oldsymbol{eta}$ | A | |
|----|-----------------------------------------------------|---|-----------------|-------------|------------------|----------------------------------------|---|
| 1. | | (| [0], | [], | $[1,\ldots,8],$ | Ø |) |
| 2. | $Shift^\lambda \Longrightarrow$ | (| [0, 1], | [], | $[2,\ldots,8],$ | Ø |) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \Longrightarrow$ | (| [0], | [1], | $[2,\ldots,8],$ | $A_1=(1,\mathtt{Atr},2)$ |) |
| 4. | $Shift^\lambda \Longrightarrow$ | (| [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1 |) |
| 5. | $No	ext{-}Arc^n \implies$ | (| [0, 1], | [2], | $[3,\ldots,8],$ | A_1 |) |
| 6. | $No	ext{-}Arc^n \implies$ | (| [0], | [1, 2], | $[3,\ldots,8],$ | A_1 |) |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \Longrightarrow$ | (| [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \mathtt{Pred}, 3)$ |) |
| 8. | $Shift^\lambda \implies$ | (| $[0,\ldots,3],$ | [] | $[4,\ldots,8],$ | A_2 |) |

| | | | λ_1 | λ_2 | β | A | |
|----|-----------------------------------------------------|---|-----------------|-------------|-----------------|----------------------------------------|---|
| 1. | | (| [0], | [], | $[1,\ldots,8],$ | Ø |) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | [], | $[2,\ldots,8],$ | Ø |) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \Longrightarrow$ | (| [0], | [1], | $[2,\ldots,8],$ | $A_1=(1,\mathtt{Atr},2)$ |) |
| 4. | $Shift^\lambda \Longrightarrow$ | (| [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1 |) |
| 5. | $No	ext{-}Arc^n \implies$ | (| [0, 1], | [2], | $[3,\ldots,8],$ | A_1 |) |
| 6. | $No	ext{-}Arc^n \implies$ | (| [0], | [1, 2], | $[3,\ldots,8],$ | A_1 |) |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \Longrightarrow$ | (| [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3)$ |) |
| 8. | Shift $^{\lambda} \Longrightarrow$ | (| $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |) |
| 9. | $Shift^\lambda \Longrightarrow$ | (| $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | A_2 |) |

Projective list-based algorithm

| | | | λ_1 | λ_2 | eta | A | |
|-----|-----------------------------------------------------|---|-----------------|-------------|-----------------|----------------------------------------|---|
| 1. | | (| [0], | [], | $[1,\ldots,8],$ | Ø |) |
| 2. | $Shift^\lambda \Longrightarrow$ | (| [0, 1], | [], | $[2,\ldots,8],$ | Ø |) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \Longrightarrow$ | (| [0], | [1], | $[2,\ldots,8],$ | $A_1=(1,\mathtt{Atr},2)$ |) |
| 4. | $Shift^\lambda \Longrightarrow$ | (| [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1 |) |
| 5. | $No	ext{-}Arc^n \implies$ | (| [0, 1], | [2], | $[3,\ldots,8],$ | A_1 |) |
| 6. | $No\text{-}Arc^n \Longrightarrow$ | (| [0], | [1, 2], | $[3,\ldots,8],$ | A_1 |) |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \Longrightarrow$ | (| [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \mathtt{Pred}, 3)$ |) |
| 8. | $Shift^\lambda \Longrightarrow$ | (| $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |) |
| 9. | $Shift^\lambda \implies$ | (| $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | A_2 |) |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies$ | (| $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | $A_3 = A_2 \cup (5, \mathtt{AuxZ}, 4)$ |) |

Projective list-based algorithm

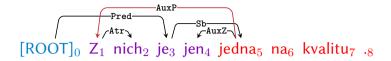


| | | | λ_1 | λ_2 | $oldsymbol{eta}$ | A | |
|-----|-----------------------------------------------------|---|-----------------|-------------|------------------|----------------------------------------|---|
| 1. | | (| [0], | [], | $[1,\ldots,8],$ | Ø |) |
| 2. | $Shift^\lambda \Longrightarrow$ | (| [0, 1], | [], | $[2,\ldots,8],$ | Ø |) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \Longrightarrow$ | (| [0], | [1], | $[2,\ldots,8],$ | $A_1=(1,\mathtt{Atr},2)$ |) |
| 4. | $Shift^\lambda \Longrightarrow$ | (| [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1 |) |
| 5. | $No	ext{-}Arc^n \implies$ | (| [0, 1], | [2], | $[3,\ldots,8],$ | A_1 |) |
| 6. | $No	ext{-}Arc^n \implies$ | (| [0], | [1, 2], | $[3,\ldots,8],$ | A_1 |) |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \Longrightarrow$ | (| [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3)$ |) |
| 8. | $Shift^\lambda \Longrightarrow$ | (| $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |) |
| 9. | $Shift^\lambda \Longrightarrow$ | (| $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | A_2 |) |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies$ | (| $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | $A_3 = A_2 \cup (5, \mathtt{AuxZ}, 4)$ |) |
| 11. | Rіght-Arc $_{\mathtt{Sb}}^{n} \Longrightarrow$ | (| [0, 1, 2], | [3, 4], | $[5,\ldots,8],$ | $A_4=A_3\cup(3,\operatorname{Sb},5)$ |) |

Projective list-based algorithm

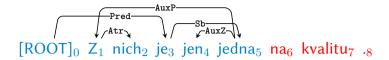
| | | | λ_1 | λ_2 | $oldsymbol{eta}$ | A | |
|-----|-----------------------------------------------------|---|-----------------|-------------|------------------|----------------------------------------|---|
| 1. | | (| [0], | [], | $[1,\ldots,8],$ | Ø |) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | [], | $[2,\ldots,8],$ | Ø |) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \Longrightarrow$ | (| [0], | [1], | $[2,\ldots,8],$ | $A_1=(1,\mathtt{Atr},2)$ |) |
| 4. | $Shift^\lambda \Longrightarrow$ | (| [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1 |) |
| 5. | $No	ext{-}Arc^n \implies$ | (| [0, 1], | [2], | $[3,\ldots,8],$ | A_1 |) |
| 6. | $No	ext{-}Arc^n \implies$ | (| [0], | [1, 2], | $[3,\ldots,8],$ | A_1 |) |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \Longrightarrow$ | (| [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \mathtt{Pred}, 3)$ |) |
| 8. | $Shift^\lambda \implies$ | (| $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |) |
| 9. | $Shift^\lambda \Longrightarrow$ | (| $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | A_2 |) |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies$ | (| $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | $A_3=A_2\cup(5,\mathtt{AuxZ},4)$ |) |
| 11. | $Right	ext{-}Arc^n_{Sb} \implies$ | (| [0, 1, 2], | [3, 4], | $[5,\ldots,8],$ | $A_4 = A_3 \cup (3, \mathtt{Sb}, 5)$ |) |
| 12. | $\operatorname{No-Arc}^n \Longrightarrow$ | (| [0, 1], | [2, 3, 4], | $[5,\ldots,8],$ | A_4 |) |

Projective list-based algorithm



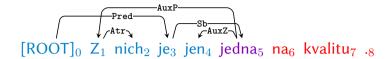
| | | | λ_1 | λ_2 | eta | A |
|-----|-----------------------------------------------------|---|-----------------|-----------------|-----------------|---------------------------------------------|
| 1. | | (| [0], | [], | $[1,\ldots,8],$ | Ø) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | [], | $[2,\ldots,8],$ | Ø) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \Longrightarrow$ | (| [0], | [1], | $[2,\ldots,8],$ | $A_1 = (1, \texttt{Atr}, 2) $ |
| 4. | $Shift^\lambda \implies$ | (| [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1) |
| 5. | $No	ext{-}Arc^n \implies$ | (| [0, 1], | [2], | $[3,\ldots,8],$ | A_1) |
| 6. | $No	ext{-}Arc^n \implies$ | (| [0], | [1, 2], | $[3,\ldots,8],$ | A_1) |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \Longrightarrow$ | (| [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3))$ |
| 8. | $Shift^\lambda \implies$ | (| $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |
| 9. | $Shift^\lambda \implies$ | (| $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | A_2 |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies$ | (| $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | $A_3 = A_2 \cup (5, \mathtt{AuxZ}, 4))$ |
| 11. | Rіснт-Arc $_{\mathtt{Sb}}^{n} \Longrightarrow$ | (| [0, 1, 2], | [3, 4], | $[5,\ldots,8],$ | $A_4 = A_3 \cup (3, \operatorname{Sb}, 5) $ |
| 12. | $No	ext{-}Arc^n \implies$ | (| [0, 1], | [2, 3, 4], | $[5,\ldots,8],$ | A_4 |
| 13. | $Left\text{-}Arc^n_{\mathtt{AuxP}} \implies$ | (| [0], | $[1,\ldots,4],$ | $[5,\ldots,8],$ | $A_5 = A_4 \cup (5, \mathtt{AuxP}, 1)$) |

Projective list-based algorithm



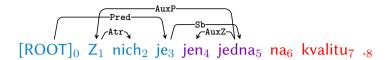
| | | | λ_1 | λ_2 | $oldsymbol{eta}$ | A | |
|-----|-----------------------------------------------------|---|-----------------|-----------------|------------------|---------------------------------------------|--|
| 1. | | (| [0], | [], | $[1,\ldots,8],$ | Ø) | |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | [], | $[2,\ldots,8],$ | Ø) | |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \implies$ | (| [0], | [1], | $[2,\ldots,8],$ | $A_1 = (1, \texttt{Atr}, 2) $ | |
| 4. | $Shift^\lambda \Longrightarrow$ | (| [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1 | |
| 5. | $No	ext{-}Arc^n \implies$ | (| [0, 1], | [2], | $[3,\ldots,8],$ | A_1 | |
| 6. | $No	ext{-}Arc^n \implies$ | (| [0], | [1, 2], | $[3,\ldots,8],$ | A_1 | |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \Longrightarrow$ | (| [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3))$ | |
| 8. | $Shift^\lambda \implies$ | (| $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 | |
| 9. | $Shift^\lambda \implies$ | (| $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | A_2 | |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies$ | (| $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | $A_3 = A_2 \cup (5, \mathtt{AuxZ}, 4))$ | |
| 11. | $Right	ext{-}Arc^n_{\mathtt{Sb}} \implies$ | (| [0, 1, 2], | [3, 4], | $[5,\ldots,8],$ | $A_4 = A_3 \cup (3, \operatorname{Sb}, 5) $ | |
| 12. | $No	ext{-}Arc^n \implies$ | (| [0, 1], | [2, 3, 4], | $[5,\ldots,8],$ | A_4 | |
| 13. | $Left\text{-}Arc^n_{\mathtt{AuxP}} \implies$ | (| [0], | $[1,\ldots,4],$ | $[5,\ldots,8],$ | $A_5 = A_4 \cup (5, \mathtt{AuxP}, 1))$ | |
| 14. | $Shift^\lambda \Longrightarrow$ | (| $[0,\ldots,5],$ | [], | [6, 7, 8], | A_5 | |

Projective list-based algorithm



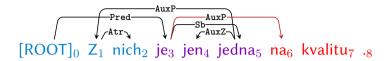
| | | | λ_1 | λ_2 | eta | A | |
|-----|-----------------------------------------------------|---|-----------------|-----------------|-----------------|----------------------------------------|---|
| 1. | | (| [0], | [], | $[1,\ldots,8],$ | Ø |) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | [], | $[2,\ldots,8],$ | Ø |) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \Longrightarrow$ | (| [0], | [1], | $[2,\ldots,8],$ | $A_1=(1,\mathtt{Atr},2)$ |) |
| 4. | $Shift^\lambda \implies$ | (| [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1 |) |
| 5. | $No	ext{-}Arc^n \implies$ | (| [0, 1], | [2], | $[3,\ldots,8],$ | A_1 |) |
| 6. | $No	ext{-}Arc^n \implies$ | (| [0], | [1, 2], | $[3,\ldots,8],$ | A_1 |) |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \Longrightarrow$ | (| [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3)$ |) |
| 8. | Shift $^{\lambda} \Longrightarrow$ | (| $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |) |
| 9. | $Shift^\lambda \implies$ | (| $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | A_2 |) |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies$ | (| $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | $A_3 = A_2 \cup (5, \mathtt{AuxZ}, 4)$ |) |
| 11. | $Right	ext{-}Arc^n_{\mathtt{Sb}} \implies$ | (| [0, 1, 2], | [3, 4], | $[5,\ldots,8],$ | $A_4=A_3\cup(3,\operatorname{Sb},5)$ |) |
| 12. | $No	ext{-}Arc^n \implies$ | (| [0, 1], | [2, 3, 4], | $[5,\ldots,8],$ | A_4 |) |
| 13. | $Left\text{-}Arc^n_{\mathtt{AuxP}} \implies$ | (| [0], | $[1,\ldots,4],$ | $[5,\ldots,8],$ | $A_5 = A_4 \cup (5, \mathtt{AuxP}, 1)$ |) |
| 14. | $Shift^\lambda \implies$ | (| $[0,\ldots,5],$ | [], | [6, 7, 8], | A_5 |) |
| 15. | $No	ext{-}Arc^n \implies$ | (| $[0,\ldots,4],$ | [5], | [6, 7, 8], | A_5 |) |

Projective list-based algorithm



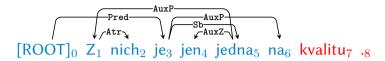
| | | | λ_1 | λ_2 | eta | A |
|-----|-----------------------------------------------------|---|-----------------|-----------------|-----------------|-----------------------------------------------|
| 1. | | (| [0], | [], | $[1,\ldots,8],$ | Ø) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | [], | $[2,\ldots,8],$ | Ø) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \Longrightarrow$ | (| [0], | [1], | $[2,\ldots,8],$ | $A_1 = (1, \texttt{Atr}, 2) $ |
| 4. | $Shift^\lambda \implies$ | (| [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1) |
| 5. | $No	ext{-}Arc^n \implies$ | (| [0, 1], | [2], | $[3,\ldots,8],$ | A_1 |
| 6. | $No	ext{-}Arc^n \implies$ | (| [0], | [1, 2], | $[3,\ldots,8],$ | A_1 |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \Longrightarrow$ | (| [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3))$ |
| 8. | $Shift^\lambda \implies$ | (| $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |
| 9. | $Shift^\lambda \implies$ | (| $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | A_2 |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies$ | (| $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | $A_3 = A_2 \cup (5, \mathtt{AuxZ}, 4))$ |
| 11. | $Right	ext{-}Arc^n_\mathtt{Sb} \implies$ | (| [0, 1, 2], | [3, 4], | $[5,\ldots,8],$ | $A_4 = A_3 \cup (3, \mathtt{Sb}, 5) \qquad)$ |
| 12. | $No	ext{-}Arc^n \implies$ | (| [0, 1], | [2, 3, 4], | $[5,\ldots,8],$ | A_4 |
| 13. | $Left\text{-}Arc^n_{\mathtt{AuxP}} \implies$ | (| [0], | $[1,\ldots,4],$ | $[5,\ldots,8],$ | $A_5 = A_4 \cup (5, \mathtt{AuxP}, 1)$) |
| 14. | $Shift^\lambda \implies$ | (| $[0,\ldots,5],$ | [], | [6, 7, 8], | A_5 |
| 15. | $No	ext{-}Arc^n \implies$ | (| $[0,\ldots,4],$ | [5], | [6, 7, 8], | A_5 |
| 16. | $No	ext{-}Arc^n \implies$ | (| $[0,\ldots,3],$ | [4, 5], | [6, 7, 8], | A_5 |

Projective list-based algorithm



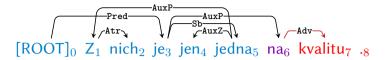
| | | | λ_1 | λ_2 | β | A |
|-----|-----------------------------------------------------|---|-----------------|-----------------|-----------------|---------------------------------------------|
| 1. | | (| [0], | [], | $[1,\ldots,8],$ | Ø) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | [], | $[2,\ldots,8],$ | Ø) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \Longrightarrow$ | (| [0], | [1], | $[2,\ldots,8],$ | $A_1 = (1, \texttt{Atr}, 2) $ |
| 4. | $Shift^\lambda \implies$ | (| [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1 |
| 5. | $No	ext{-}Arc^n \implies$ | (| [0, 1], | [2], | $[3,\ldots,8],$ | A_1) |
| 6. | $No	ext{-}Arc^n \implies$ | (| [0], | [1, 2], | $[3,\ldots,8],$ | A_1) |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \Longrightarrow$ | (| [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3))$ |
| 8. | $Shift^\lambda \implies$ | (| $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |
| 9. | $Shift^\lambda \implies$ | (| $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | A_2 |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies$ | (| $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | $A_3 = A_2 \cup (5, \mathtt{AuxZ}, 4))$ |
| 11. | $Right	ext{-}Arc^n_\mathtt{Sb} \implies$ | (| [0, 1, 2], | [3, 4], | $[5,\ldots,8],$ | $A_4 = A_3 \cup (3, \operatorname{Sb}, 5) $ |
| 12. | $No	ext{-}Arc^n \implies$ | (| [0, 1], | [2, 3, 4], | $[5,\ldots,8],$ | A_4 |
| 13. | $Left\text{-}Arc^n_{\mathtt{AuxP}} \implies$ | (| [0], | $[1,\ldots,4],$ | $[5,\ldots,8],$ | $A_5 = A_4 \cup (5, \mathtt{AuxP}, 1)$) |
| 14. | $Shift^\lambda \implies$ | (| $[0,\ldots,5],$ | [], | [6, 7, 8], | A_5 |
| 15. | $No	ext{-}Arc^n \implies$ | (| $[0,\ldots,4],$ | [5], | [6, 7, 8], | A_5 |
| 16. | $No	ext{-}Arc^n \implies$ | (| $[0,\ldots,3],$ | [4, 5], | [6, 7, 8], | A_5 |
| 17. | $Right	ext{-}Arc^n_{\mathtt{AuxP}} \implies$ | (| [0, 1, 2], | [3, 4, 5], | [6, 7, 8], | $A_6 = A_5 \cup (3, \mathtt{AuxP}, 6))$ |

Projective list-based algorithm



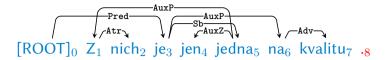
| | | λ_1 | λ_2 | β | A |
|-----|----------------------------------------------------------|-----------------|-----------------|-----------------|---------------------------------------------|
| 1. | (| [0], | [], | $[1,\ldots,8],$ | Ø) |
| 2. | $Shift^\lambda \implies \ ($ | [0, 1], | [], | $[2,\ldots,8],$ | \varnothing |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \Longrightarrow ($ | [0], | [1], | $[2,\ldots,8],$ | $A_1 = (1, \texttt{Atr}, 2) $ |
| 4. | $Shift^\lambda \implies \ ($ | [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1) |
| 5. | $No	ext{-}Arc^n \implies ($ | [0, 1], | [2], | $[3,\ldots,8],$ | A_1) |
| 6. | $No	ext{-}Arc^n \implies ($ | [0], | [1, 2], | $[3,\ldots,8],$ | A_1) |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \implies ($ | [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3))$ |
| 8. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |
| 9. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | A_2 |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies ($ | $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | $A_3 = A_2 \cup (5, \mathtt{AuxZ}, 4))$ |
| 11. | $Right\text{-}Arc^n_{\mathtt{Sb}} \implies ($ | [0, 1, 2], | [3, 4], | $[5,\ldots,8],$ | $A_4 = A_3 \cup (3, \operatorname{Sb}, 5) $ |
| 12. | $No	ext{-}Arc^n \implies ($ | [0, 1], | [2, 3, 4], | $[5,\ldots,8],$ | A_4 |
| 13. | $Left\text{-}Arc^n_{\mathtt{Aux}\mathtt{P}} \implies ($ | [0], | $[1,\ldots,4],$ | $[5,\ldots,8],$ | $A_5 = A_4 \cup (5, \mathtt{AuxP}, 1)$) |
| 14. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,5],$ | [], | [6, 7, 8], | A_5 |
| 15. | $No	ext{-}Arc^n \implies ($ | $[0,\ldots,4],$ | [5], | [6, 7, 8], | A_5 |
| 16. | $No	ext{-}Arc^n \implies ($ | $[0,\ldots,3],$ | [4, 5], | [6, 7, 8], | A_5 |
| 17. | $Right\text{-}Arc^n_{\mathtt{Aux}\mathtt{P}} \implies ($ | [0, 1, 2], | [3, 4, 5], | [6, 7, 8], | $A_6=A_5\cup(3,\mathtt{AuxP},6)$) |
| 18. | $Shift^\lambda \implies \ ($ | $[0,\ldots,6],$ | [], | [7, 8], | A_6 |

Projective list-based algorithm



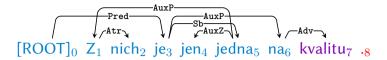
| | | λ_1 | λ_2 | β | A |
|-----|------------------------------------------------------|-----------------|-----------------|-----------------|-----------------------------------------------|
| 1. | (| [0], | [], | $[1,\ldots,8],$ | Ø) |
| 2. | $Shift^\lambda \Longrightarrow ($ | [0, 1], | [], | $[2,\ldots,8],$ | \varnothing |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \Longrightarrow$ (| [0], | [1], | $[2,\ldots,8],$ | $A_1 = (1, \texttt{Atr}, 2) $ |
| 4. | $Shift^\lambda \Longrightarrow ($ | [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1) |
| 5. | $No-Arc^n \implies ($ | [0, 1], | [2], | $[3,\ldots,8],$ | A_1) |
| 6. | $No-Arc^n \implies ($ | [0], | [1, 2], | $[3,\ldots,8],$ | A_1) |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \implies ($ | [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3))$ |
| 8. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |
| 9. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | A_2 |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies ($ | $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | $A_3 = A_2 \cup (5, \mathtt{AuxZ}, 4))$ |
| 11. | $Right	ext{-}Arc^n_{\mathtt{Sb}} \implies ($ | [0, 1, 2], | [3, 4], | $[5,\ldots,8],$ | $A_4 = A_3 \cup (3, \operatorname{Sb}, 5) $ |
| 12. | $No\text{-}Arc^n \implies ($ | [0, 1], | [2, 3, 4], | $[5,\ldots,8],$ | A_4 |
| 13. | $Left\text{-}Arc^n_{\mathtt{AuxP}} \implies ($ | [0], | $[1,\ldots,4],$ | $[5,\ldots,8],$ | $A_5 = A_4 \cup (5, \mathtt{AuxP}, 1)$) |
| 14. | $Shift^\lambda \implies ($ | $[0,\ldots,5],$ | [], | [6, 7, 8], | A_5 |
| 15. | $No-Arc^n \implies ($ | $[0,\ldots,4],$ | [5], | [6, 7, 8], | A_5 |
| 16. | $No-Arc^n \implies ($ | $[0,\ldots,3],$ | [4, 5], | [6, 7, 8], | A_5 |
| 17. | $Right-Arc^n_{AuxP} \Longrightarrow ($ | [0, 1, 2], | [3, 4, 5], | [6, 7, 8], | $A_6 = A_5 \cup (3, \mathtt{AuxP}, 6)$) |
| 18. | $Shift^\lambda \implies ($ | $[0,\ldots,6],$ | [], | [7, 8], | A_6 |
| 19. | $Right	ext{-}Arc^n_{\mathtt{Adv}} \Longrightarrow$ (| $[0,\ldots,5],$ | [6], | [7, 8], | $A_7 = A_6 \cup (6, \mathtt{Adv}, 7) \qquad)$ |

Projective list-based algorithm

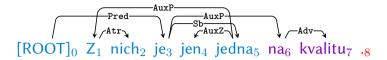


| | | λ_1 | λ_2 | β | A |
|-----|---------------------------------------------------------|-----------------|-----------------|-----------------|-----------------------------------------------|
| 1. | (| [0], | [], | $[1,\ldots,8],$ | Ø) |
| 2. | $SHIFT^\lambda \Longrightarrow ($ | [0, 1], | [], | $[2,\ldots,8],$ | Ø) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \implies ($ | [0], | [1], | $[2,\ldots,8],$ | $A_1 = (1, \texttt{Atr}, 2) $ |
| 4. | $Shift^\lambda \Longrightarrow ($ | [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1) |
| 5. | $No-Arc^n \implies ($ | [0, 1], | [2], | $[3,\ldots,8],$ | A_1 |
| 6. | $No-Arc^n \implies ($ | [0], | [1, 2], | $[3,\ldots,8],$ | A_1 |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \implies ($ | [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3))$ |
| 8. | $SHIFT^\lambda \Longrightarrow ($ | $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |
| 9. | $SHIFT^\lambda \Longrightarrow ($ | $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | A_2 |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies ($ | $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | $A_3 = A_2 \cup (5, \mathtt{AuxZ}, 4))$ |
| 11. | $Right	ext{-}Arc^n_{\mathtt{Sb}} \implies ($ | [0, 1, 2], | [3, 4], | $[5,\ldots,8],$ | $A_4 = A_3 \cup (3, \operatorname{Sb}, 5) $ |
| 12. | $No\text{-}Arc^n \implies ($ | [0, 1], | [2, 3, 4], | $[5,\ldots,8],$ | A_4 |
| 13. | $Left\text{-}Arc^n_{\mathtt{Aux}\mathtt{P}} \implies ($ | [0], | $[1,\ldots,4],$ | $[5,\ldots,8],$ | $A_5 = A_4 \cup (5, \mathtt{AuxP}, 1))$ |
| 14. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,5],$ | [], | [6, 7, 8], | A_5 |
| 15. | $No-Arc^n \implies ($ | $[0,\ldots,4],$ | [5], | [6, 7, 8], | A_5 |
| 16. | $No\text{-}Arc^n \Longrightarrow ($ | | [4, 5], | [6, 7, 8], | A_5 |
| 17. | $Right-Arc^n_{AuxP} \Longrightarrow ($ | [0, 1, 2], | [3, 4, 5], | [6, 7, 8], | $A_6 = A_5 \cup (3, \mathtt{AuxP}, 6))$ |
| 18. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,6],$ | [], | [7, 8], | A_6 |
| 19. | $Right-Arc^n_{Adv} \Longrightarrow ($ | $[0,\ldots,5],$ | [6], | [7, 8], | $A_7 = A_6 \cup (6, \mathtt{Adv}, 7) \qquad)$ |
| 20. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,7],$ | [], | [8], | A_7 |

Projective list-based algorithm

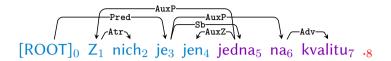


| | | λ_1 | λ_2 | β | A |
|-----|---------------------------------------------------------|-----------------|-----------------|-----------------|-----------------------------------------------|
| 1. | (| [0], | [], | $[1,\ldots,8],$ | Ø) |
| 2. | $Shift^\lambda \Longrightarrow ($ | [0, 1], | [], | $[2,\ldots,8],$ | Ø) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \Longrightarrow$ (| [0], | [1], | $[2,\ldots,8],$ | $A_1 = (1, \texttt{Atr}, 2) $ |
| 4. | $Shift^\lambda \implies ($ | [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1) |
| 5. | $No-Arc^n \implies ($ | [0, 1], | [2], | $[3,\ldots,8],$ | A_1) |
| 6. | $No	ext{-}Arc^n \implies ($ | [0], | [1, 2], | $[3,\ldots,8],$ | A_1) |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \implies ($ | [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3))$ |
| 8. | $SHIFT^\lambda \implies ($ | $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |
| 9. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | A_2 |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies ($ | $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | $A_3 = A_2 \cup (5, \mathtt{AuxZ}, 4))$ |
| 11. | $Right	ext{-}Arc^n_{\mathtt{Sb}} \implies ($ | [0, 1, 2], | [3, 4], | $[5,\ldots,8],$ | $A_4 = A_3 \cup (3, \operatorname{Sb}, 5) $ |
| 12. | $No\text{-}Arc^n \implies ($ | [0, 1], | [2, 3, 4], | $[5,\ldots,8],$ | A_4 |
| 13. | $Left\text{-}Arc^n_{\mathtt{AuxP}} \implies ($ | [0], | $[1,\ldots,4],$ | $[5,\ldots,8],$ | $A_5 = A_4 \cup (5, \mathtt{AuxP}, 1))$ |
| 14. | $Shift^\lambda \implies ($ | $[0,\ldots,5],$ | [], | [6, 7, 8], | A_5 |
| 15. | $No-Arc^n \implies ($ | $[0,\ldots,4],$ | [5], | [6, 7, 8], | A_5 |
| 16. | $No\text{-}Arc^n \implies ($ | $[0,\ldots,3],$ | [4, 5], | [6, 7, 8], | A_5 |
| 17. | $Right	ext{-}Arc^n_{\mathtt{Aux}\mathtt{P}} \implies ($ | [0, 1, 2], | [3, 4, 5], | [6, 7, 8], | $A_6 = A_5 \cup (3, \mathtt{AuxP}, 6))$ |
| 18. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,6],$ | [], | [7, 8], | A_6 |
| 19. | $Right	ext{-}Arc^n_{\mathtt{Ady}} \implies ($ | $[0,\ldots,5],$ | [6], | [7, 8], | $A_7 = A_6 \cup (6, \mathtt{Adv}, 7) \qquad)$ |
| 20. | $Shift^\lambda \Longrightarrow \ ($ | $[0,\ldots,7],$ | [], | [8], | A_7 |
| 21. | $No-Arc^n \implies ($ | $[0,\ldots,6],$ | [7], | [8], | A_7 |

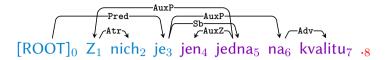


| | | λ_1 | λ_2 | β | A |
|-----|-----------------------------------------------------------------------|-----------------|-----------------|-----------------|-----------------------------------------------|
| 1. | (| [0], | [], | $[1,\ldots,8],$ | Ø) |
| 2. | $Shift^\lambda \Longrightarrow ($ | [0, 1], | [], | $[2,\ldots,8],$ | Ø) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \implies ($ | [0], | [1], | $[2,\ldots,8],$ | $A_1 = (1, \texttt{Atr}, 2) $ |
| 4. | $Shift^\lambda \Longrightarrow ($ | [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1 |
| 5. | $No-Arc^n \implies ($ | [0, 1], | [2], | $[3,\ldots,8],$ | A_1 |
| 6. | $No-Arc^n \implies ($ | [0], | [1, 2], | $[3,\ldots,8],$ | A_1 |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \implies ($ | [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3))$ |
| 8. | $SHIFT^\lambda \Longrightarrow ($ | $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |
| 9. | $Shift^\lambda \implies \ ($ | $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | A_2 |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies ($ | $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | $A_3 = A_2 \cup (5, \mathtt{AuxZ}, 4))$ |
| 11. | $Right	ext{-}Arc^n_\mathtt{Sb} \implies ($ | [0, 1, 2], | [3, 4], | $[5,\ldots,8],$ | |
| 12. | $No-Arc^n \implies ($ | [0, 1], | [2, 3, 4], | $[5,\ldots,8],$ | A_4 |
| 13. | $Left\text{-}Arc^n_{\mathtt{Aux} \overset{\mathtt{P}}{P}} \implies ($ | [0], | $[1,\ldots,4],$ | $[5,\ldots,8],$ | $A_5 = A_4 \cup (5, \mathtt{AuxP}, 1))$ |
| 14. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,5],$ | [], | [6, 7, 8], | A_5 |
| 15. | $No	ext{-}Arc^n \implies ($ | $[0,\ldots,4],$ | [5], | [6, 7, 8], | A_5 |
| 16. | $No	ext{-}Arc^n \implies ($ | $[0,\ldots,3],$ | [4, 5], | [6, 7, 8], | A_5 |
| 17. | $Right\text{-}Arc^n_{\mathtt{Aux},\mathtt{P}} \implies ($ | [0, 1, 2], | [3, 4, 5], | [6, 7, 8], | $A_6 = A_5 \cup (3, \mathtt{AuxP}, 6))$ |
| 18. | $Shift^\lambda \implies \ ($ | $[0,\ldots,6],$ | [], | [7, 8], | A_6 |
| 19. | $Right	ext{-}Arc^n_{\mathtt{Adv}} \implies ($ | $[0,\ldots,5],$ | [6], | [7, 8], | $A_7 = A_6 \cup (6, \mathtt{Adv}, 7) \qquad)$ |
| 20. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,7],$ | [], | [8], | A_7 |
| 21. | $No	ext{-}Arc^n \implies ($ | $[0,\ldots,6],$ | [7], | [8], | A_7 |
| 22. | $No	ext{-}Arc^n \implies ($ | $[0,\ldots,5],$ | [6, 7], | [8], | A_7 |

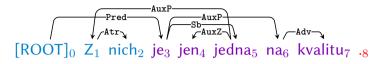
Projective list-based algorithm



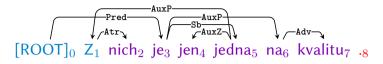
| | | λ_1 | λ_2 | eta | A |
|-----|----------------------------------------------------------|-----------------|-----------------|-----------------|-----------------------------------------------|
| 1. | (| [0], | [], | $[1,\ldots,8],$ | Ø) |
| 2. | $Shift^\lambda \Longrightarrow ($ | [0, 1], | [], | $[2,\ldots,8],$ | \varnothing |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \Longrightarrow$ (| [0], | [1], | $[2,\ldots,8],$ | $A_1 = (1, \texttt{Atr}, 2) $ |
| 4. | $Shift^\lambda \implies ($ | [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1 |
| 5. | $No-Arc^n \implies ($ | [0, 1], | [2], | $[3,\ldots,8],$ | A_1) |
| 6. | $No	ext{-}Arc^n \implies ($ | [0], | [1, 2], | $[3,\ldots,8],$ | A_1 |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \implies ($ | [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3))$ |
| 8. | $Shift^{\lambda} \Longrightarrow ($ | $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |
| 9. | $Shift^\lambda \implies ($ | $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies ($ | $[0,\ldots,3],$ | [4], | | $A_3 = A_2 \cup (5, \mathtt{AuxZ}, 4))$ |
| 11. | $Right	ext{-}Arc^n_{\mathtt{Sb}} \implies ($ | [0, 1, 2], | [3, 4], | | $A_4 = A_3 \cup (3, \operatorname{Sb}, 5) $ |
| 12. | $No\text{-}Arc^n \implies ($ | [0, 1], | [2, 3, 4], | $[5,\ldots,8],$ | A_4 |
| 13. | $Left\text{-}Arc^n_{\mathtt{AuxP}} \implies ($ | [0], | $[1,\ldots,4],$ | $[5,\ldots,8],$ | $A_5 = A_4 \cup (5, \mathtt{AuxP}, 1))$ |
| 14. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,5],$ | [], | [6, 7, 8], | A_5) |
| 15. | $No\text{-}Arc^n \implies ($ | $[0,\ldots,4],$ | [5], | [6, 7, 8], | A_5 |
| 16. | $No\text{-}Arc^n \implies ($ | $[0,\ldots,3],$ | [4, 5], | [6, 7, 8], | A_5 |
| 17. | $Right\text{-}Arc^n_{\mathtt{Aux}\mathtt{P}} \implies ($ | [0, 1, 2], | [3, 4, 5], | [6, 7, 8], | $A_6 = A_5 \cup (3, \mathtt{AuxP}, 6))$ |
| 18. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,6],$ | [], | [7, 8], | A_6 |
| 19. | $Right	ext{-}Arc^n_{\mathtt{Ady}} \Longrightarrow ($ | $[0,\ldots,5],$ | [6], | [7, 8], | $A_7 = A_6 \cup (6, \mathtt{Adv}, 7) \qquad)$ |
| 20. | $Shift^\lambda \implies ($ | $[0,\ldots,7],$ | [], | [8], | A_7 |
| 21. | $No\text{-}Arc^n \implies ($ | $[0,\ldots,6],$ | [7], | [8], | A_7 |
| 22. | $No\text{-}Arc^n \implies ($ | $[0,\ldots,5],$ | [6, 7], | [8], | A_7 |
| 23. | $No\text{-}Arc^n \implies ($ | $[0,\ldots,4],$ | [5, 6, 7], | [8], | A_7 |



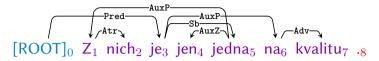
| | | λ_1 | λ_2 | β | A |
|-----|---------------------------------------------------------|-----------------|-----------------|-----------------|---------------------------------------------|
| 1. | (| [0], | [], | $[1,\ldots,8],$ | Ø) |
| 2. | $Shift^\lambda \Longrightarrow ($ | [0, 1], | [], | $[2,\ldots,8],$ | \varnothing |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \Longrightarrow$ (| [0], | [1], | $[2,\ldots,8],$ | $A_1 = (1, \texttt{Atr}, 2) $ |
| 4. | $Shift^\lambda \implies ($ | [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1 |
| 5. | $No\text{-}Arc^n \implies ($ | [0, 1], | [2], | $[3,\ldots,8],$ | A_1 |
| 6. | $No	ext{-}Arc^n \implies ($ | [0], | [1, 2], | $[3,\ldots,8],$ | |
| 7. | $Right\text{-}Arc^n_{\mathtt{Pred}} \implies ($ | [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3))$ |
| 8. | $Shift^{\lambda} \Longrightarrow ($ | $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |
| 9. | $Shift^\lambda \implies ($ | $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | A_2 |
| 10. | nuxu | $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | $A_3 = A_2 \cup (5, \mathtt{AuxZ}, 4))$ |
| 11. | $Right	ext{-}Arc^n_{\mathtt{Sb}} \implies ($ | [0, 1, 2], | [3, 4], | $[5,\ldots,8],$ | $A_4 = A_3 \cup (3, \operatorname{Sb}, 5) $ |
| 12. | $No\text{-}Arc^n \implies ($ | [0, 1], | [2, 3, 4], | $[5,\ldots,8],$ | |
| 13. | $Left\text{-}Arc^n_{\mathtt{Aux}\mathtt{P}} \implies ($ | | $[1,\ldots,4],$ | $[5,\ldots,8],$ | $A_5 = A_4 \cup (5, \mathtt{AuxP}, 1))$ |
| 14. | $Shift^\lambda \implies ($ | $[0,\ldots,5],$ | [], | [6, 7, 8], | A_5 |
| 15. | $No	ext{-}Arc^n \implies ($ | | [5], | [6, 7, 8], | A_5 |
| 16. | $No\text{-}Arc^n \Longrightarrow ($ | | [4, 5], | [6, 7, 8], | A_5 |
| 17. | $Right-Arc^n_{AuxP} \Longrightarrow ($ | [0, 1, 2], | [3, 4, 5], | [6, 7, 8], | $A_6 = A_5 \cup (3, \mathtt{AuxP}, 6))$ |
| 18. | $Shift^\lambda \implies ($ | $[0,\ldots,6],$ | [], | [7, 8], | A_6 |
| 19. | $Right	ext{-}Arc^n_{\mathtt{Adv}} \implies ($ | $[0,\ldots,5],$ | [6], | [7, 8], | $A_7 = A_6 \cup (6, Adv, 7) \qquad)$ |
| 20. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,7],$ | [], | [8], | A_7 |
| 21. | $No	ext{-}Arc^n \implies ($ | $[0,\ldots,6],$ | [7], | [8], | A_7 |
| 22. | $No-Arc^n \Longrightarrow ($ | $[0,\ldots,5],$ | [6, 7], | [8], | A_7 |
| 23. | $No-Arc^n \Longrightarrow ($ | $[0,\ldots,4],$ | [5, 6, 7], | [8], | A_7 |
| 24. | $No	ext{-}Arc^n \implies ($ | $[0,\ldots,3],$ | $[4,\ldots,7],$ | [8], | A_7 |



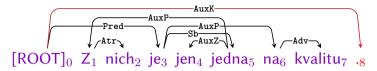
| | | λ_1 | λ_2 | β | A |
|-----|---------------------------------------------------------|-----------------|-----------------|-----------------|-----------------------------------------------|
| 1. | (| [0], | [], | $[1,\ldots,8],$ | Ø) |
| 2. | $SHIFT^\lambda \Longrightarrow ($ | [0, 1], | <u> </u> | $[2,\ldots,8],$ | Ø) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \Longrightarrow$ (| [0], | [1], | $[2,\ldots,8],$ | $A_1 = (1, \texttt{Atr}, 2) $ |
| 4. | $SHIFT^\lambda \Longrightarrow ($ | [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1) |
| 5. | $No-Arc^n \implies ($ | [0, 1], | [2], | $[3,\ldots,8],$ | A_1) |
| 6. | $No\text{-}Arc^n \implies ($ | [0], | [1, 2], | $[3,\ldots,8],$ | |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \implies ($ | [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3))$ |
| 8. | $SHIFT^{\lambda} \Longrightarrow ($ | $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |
| 9. | $SHIFT^\lambda \implies ($ | $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | , |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies ($ | $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | |
| 11. | $Right-Arc^n_{Sb} \Longrightarrow ($ | [0, 1, 2], | [3, 4], | | $A_4 = A_3 \cup (3, \mathtt{Sb}, 5) $ |
| 12. | $No-Arc^n \implies ($ | [0, 1], | [2, 3, 4], | | |
| 13. | $Left\text{-}Arc^n_{\mathtt{Aux}\mathtt{P}} \implies ($ | | $[1,\ldots,4],$ | $[5,\ldots,8],$ | $A_5 = A_4 \cup (5, \mathtt{AuxP}, 1))$ |
| 14. | $SHIFT^\lambda \implies ($ | $[0,\ldots,5],$ | [], | [6, 7, 8], | A_5) |
| 15. | $No-Arc^n \implies ($ | $[0,\ldots,4],$ | [5], | [6, 7, 8], | A_5) |
| 16. | $No-Arc^n \implies ($ | | [4, 5], | [6, 7, 8], | A_5 |
| 17. | $Right-Arc^n_{Aux^p} \Longrightarrow ($ | [0, 1, 2], | [3, 4, 5], | [6, 7, 8], | $A_6 = A_5 \cup (3, \mathtt{AuxP}, 6))$ |
| 18. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,6],$ | [], | [7, 8], | A_6 |
| 19. | $Right	ext{-}Arc^n_{\mathtt{Adv}} \implies ($ | $[0,\ldots,5],$ | [6], | [7, 8], | $A_7 = A_6 \cup (6, \mathtt{Adv}, 7) \qquad)$ |
| 20. | $SHIFT^\lambda \implies ($ | $[0,\ldots,7],$ | [], | [8], | A_7 |
| 21. | $No-Arc^n \implies ($ | $[0,\ldots,6],$ | [7], | [8], | A_7 |
| 22. | $No-Arc^n \Longrightarrow ($ | | [6, 7], | [8], | A_7 |
| 23. | $No-Arc^n \Longrightarrow ($ | $[0,\ldots,4],$ | [5, 6, 7], | [8], | A_7 |
| 24. | $No-Arc^n \Longrightarrow ($ | $[0,\ldots,3],$ | $[4,\ldots,7],$ | [8], | A_7 |
| 25. | $No\text{-}Arc^n \implies ($ | [0, 1, 2], | $[3,\ldots,7],$ | [8], | A_7 |



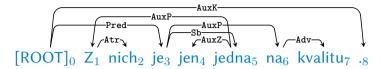
| | | λ_1 | λ_2 | β | A |
|-----|-----------------------------------------------------------|-----------------|-----------------|-----------------|-----------------------------------------------|
| 1. | (| [0], | [], | $[1,\ldots,8],$ | Ø) |
| 2. | $Shift^\lambda \Longrightarrow ($ | [0, 1], | [], | $[2,\ldots,8],$ | Ø) |
| 3. | $Right	ext{-}Arc^n_\mathtt{Atr} \implies ($ | [0], | [1], | $[2,\ldots,8],$ | $A_1 = (1, \mathtt{Atr}, 2) $ |
| 4. | $Shift^\lambda \Longrightarrow ($ | [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1 |
| 5. | $No-Arc^n \implies ($ | [0, 1], | [2], | $[3,\ldots,8],$ | A_1) |
| 6. | $No	ext{-}Arc^n \implies ($ | [0], | [1, 2], | $[3,\ldots,8],$ | * |
| 7. | $Right\text{-}Arc^n_{\mathtt{Pred}} \implies ($ | [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3))$ |
| 8. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |
| 9. | $Shift^\lambda \implies ($ | $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies ($ | $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | |
| 11. | $Right	ext{-}Arc^n_{\mathtt{Sb}} \implies ($ | [0, 1, 2], | [3, 4], | $[5,\ldots,8],$ | $A_4 = A_3 \cup (3, \operatorname{Sb}, 5) $ |
| 12. | $No\text{-}Arc^n \implies ($ | [0, 1], | [2, 3, 4], | $[5,\ldots,8],$ | |
| 13. | $Left\text{-}Arc^n_{\mathtt{Aux}\mathtt{P}} \implies ($ | | $[1,\ldots,4],$ | $[5,\ldots,8],$ | $A_5 = A_4 \cup (5, \mathtt{AuxP}, 1)$) |
| 14. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,5],$ | [], | [6, 7, 8], | A_5 |
| 15. | $No	ext{-}Arc^n \implies ($ | $[0,\ldots,4],$ | [5], | [6, 7, 8], | A_5 |
| 16. | $No	ext{-}Arc^n \implies ($ | $[0,\ldots,3],$ | | [6, 7, 8], | A_5) |
| 17. | $Right\text{-}Arc^n_{\mathtt{Aux},\mathtt{P}} \implies ($ | | [3, 4, 5], | [6, 7, 8], | $A_6 = A_5 \cup (3, \mathtt{AuxP}, 6))$ |
| 18. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,6],$ | [], | [7, 8], | A_6 |
| 19. | $Right	ext{-}Arc^n_{\mathtt{Adv}} \implies ($ | $[0,\ldots,5],$ | [6], | [7, 8], | $A_7 = A_6 \cup (6, \mathtt{Adv}, 7) \qquad)$ |
| 20. | $Shift^\lambda \implies ($ | $[0,\ldots,7],$ | [], | [8], | A_7 |
| 21. | $No	ext{-}Arc^n \implies ($ | $[0,\ldots,6],$ | [7], | [8], | A_7 |
| 22. | $No-Arc^n \implies ($ | | [6, 7], | [8], | A_7 |
| 23. | $No-Arc^n \implies ($ | | [5, 6, 7], | [8], | A_7 |
| 24. | $No-Arc^n \Longrightarrow ($ | $[0,\ldots,3],$ | $[4,\ldots,7],$ | [8], | A_7 |
| 25. | $No-Arc^n \Longrightarrow ($ | [0, 1, 2], | $[3,\ldots,7],$ | [8], | A_7 |
| 26. | $No-Arc^n \implies ($ | [0, 1], | $[2,\ldots,7],$ | [8], | A_7 |



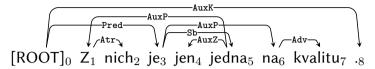
| | | \ | ١ | β | \overline{A} |
|-----|----------------------------------------------------------|-----------------|-----------------|-----------------|-----------------------------------------------|
| | | λ_1 | λ_2 | ρ | A |
| 1. | (| [0], | [], | $[1,\ldots,8],$ | \varnothing |
| 2. | $SHIFT^\lambda \Longrightarrow ($ | [0, 1], | [], | $[2,\ldots,8],$ | \varnothing |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \Longrightarrow$ (| [0], | [1], | $[2,\ldots,8],$ | $A_1 = (1, \texttt{Atr}, 2) $ |
| 4. | $SHIFT^\lambda \Longrightarrow ($ | [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1 |
| 5. | $No-Arc^n \implies ($ | [0, 1], | [2], | $[3,\ldots,8],$ | A_1 |
| 6. | $No-Arc^n \implies ($ | [0], | [1, 2], | $[3,\ldots,8],$ | A_1 |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \implies ($ | [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3))$ |
| 8. | $SHIFT^{\lambda} \Longrightarrow ($ | $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |
| 9. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | , |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies ($ | $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | |
| 11. | $Right	ext{-}Arc^n_{\mathtt{Sb}} \implies ($ | [0, 1, 2], | [3, 4], | $[5,\ldots,8],$ | $A_4 = A_3 \cup (3, \operatorname{Sb}, 5) $ |
| 12. | $No-Arc^n \implies ($ | [0, 1], | [2, 3, 4], | $[5,\ldots,8],$ | |
| 13. | $Left\text{-}Arc^n_{\mathtt{Aux}\mathtt{P}} \implies ($ | [0], | $[1,\ldots,4],$ | $[5,\ldots,8],$ | $A_5 = A_4 \cup (5, \mathtt{AuxP}, 1))$ |
| 14. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,5],$ | [], | [6, 7, 8], | A_5 |
| 15. | $No-Arc^n \implies ($ | $[0,\ldots,4],$ | [5], | [6, 7, 8], | A_5 |
| 16. | $No\text{-}Arc^n \implies ($ | $[0,\ldots,3],$ | [4, 5], | [6, 7, 8], | A_5 |
| 17. | $Right\text{-}Arc^n_{\mathtt{Aux}\mathtt{P}} \implies ($ | [0, 1, 2], | [3, 4, 5], | [6, 7, 8], | $A_6 = A_5 \cup (3, \mathtt{AuxP}, 6))$ |
| 18. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,6],$ | [], | [7, 8], | A_6 |
| 19. | $Right\text{-}Arc^n_{\mathtt{Adv}} \implies ($ | $[0,\ldots,5],$ | [6], | [7, 8], | $A_7 = A_6 \cup (6, \mathtt{Adv}, 7) \qquad)$ |
| 20. | $Shift^\lambda \implies ($ | $[0,\ldots,7],$ | [], | [8], | A_7 |
| 21. | $No\text{-}Arc^n \implies ($ | $[0,\ldots,6],$ | [7], | [8], | A_7 |
| 22. | $No-Arc^n \implies ($ | $[0,\ldots,5],$ | [6, 7], | [8], | A_7 |
| 23. | $No	ext{-}Arc^n \implies ($ | | [5, 6, 7], | [8], | A_7 |
| 24. | $No-Arc^n \implies ($ | $[0,\ldots,3],$ | $[4,\ldots,7],$ | [8], | A_7 |
| 25. | $No\text{-}Arc^n \Longrightarrow ($ | [0, 1, 2], | $[3,\ldots,7],$ | [8], | A_7 |
| 26. | $No\text{-}Arc^n \implies ($ | [0, 1], | $[2,\ldots,7],$ | [8], | A_7 |
| 27. | $No\text{-}Arc^n \implies ($ | [0], | $[1,\ldots,7],$ | [8], | A_7 |



| | | λ_1 | λ_2 | β | A |
|-----|----------------------------------------------------------|-----------------|-----------------|-------------------|-----------------------------------------------|
| 1. | (| [0], | [], | $[1,\ldots,8],$ | Ø) |
| 2. | $Shift^\lambda \Longrightarrow ($ | [0, 1], | [], | $[2, \ldots, 8],$ | Ø) |
| 3. | $Right	ext{-}Arc^n_\mathtt{Atr} \implies ($ | [0], | [1], | $[2,\ldots,8],$ | $A_1 = (1, \texttt{Atr}, 2) $ |
| 4. | $SHIFT^\lambda \Longrightarrow ($ | [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1) |
| 5. | $No-Arc^n \implies ($ | [0, 1], | [2], | $[3,\ldots,8],$ | A_1) |
| 6. | $No-Arc^n \implies ($ | [0], | [1, 2], | $[3,\ldots,8],$ | A_1) |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \implies ($ | [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \mathtt{Pred}, 3))$ |
| 8. | $SHIFT^\lambda \implies ($ | $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |
| 9. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | A_2 |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies ($ | $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | $A_3 = A_2 \cup (5, \mathtt{AuxZ}, 4))$ |
| 11. | $Right-Arc^n_{Sb} \Longrightarrow ($ | [0, 1, 2], | [3, 4], | $[5,\ldots,8],$ | $A_4 = A_3 \cup (3, \operatorname{Sb}, 5) $ |
| 12. | $No-Arc^n \implies ($ | [0, 1], | [2, 3, 4], | $[5,\ldots,8],$ | A_4 |
| 13. | $Left\text{-}Arc^n_{\mathtt{AuxP}} \implies ($ | [0], | $[1,\ldots,4],$ | $[5,\ldots,8],$ | $A_5 = A_4 \cup (5, \mathtt{AuxP}, 1))$ |
| 14. | $SHIFT^\lambda \implies ($ | $[0,\ldots,5],$ | [], | [6, 7, 8], | A_5 |
| 15. | $No	ext{-}Arc^n \implies ($ | $[0,\ldots,4],$ | [5], | [6, 7, 8], | A_5 |
| 16. | $No	ext{-}Arc^n \implies ($ | $[0,\ldots,3],$ | [4, 5], | [6, 7, 8], | A_5 |
| 17. | $Right\text{-}Arc^n_{\mathtt{Aux}\mathtt{P}} \implies ($ | [0, 1, 2], | [3, 4, 5], | [6, 7, 8], | $A_6 = A_5 \cup (3, \mathtt{AuxP}, 6)$) |
| 18. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,6],$ | [], | [7, 8], | A_6 |
| 19. | $Right	ext{-}Arc^n_{\mathtt{Ady}} \Longrightarrow ($ | $[0,\ldots,5],$ | [6], | [7, 8], | $A_7 = A_6 \cup (6, \mathtt{Adv}, 7) \qquad)$ |
| 20. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,7],$ | [], | [8], | A_7 |
| 21. | $No	ext{-}Arc^n \implies ($ | $[0,\ldots,6],$ | [7], | [8], | A_7 |
| 22. | $No	ext{-}Arc^n \implies ($ | $[0,\ldots,5],$ | [6, 7], | [8], | A_7 |
| 23. | $No	ext{-}Arc^n \implies ($ | $[0,\ldots,4],$ | [5, 6, 7], | [8], | A_7 |
| 24. | $No	ext{-}Arc^n \implies ($ | $[0,\ldots,3],$ | $[4,\ldots,7],$ | [8], | A_7 |
| 25. | $No	ext{-}Arc^n \implies ($ | [0, 1, 2], | $[3,\ldots,7],$ | [8], | A_7 |
| 26. | $No	ext{-}Arc^n \implies ($ | [0, 1], | $[2,\ldots,7],$ | [8], | A_7 |
| 27. | $No	ext{-}Arc^n \implies ($ | [0], | $[1,\ldots,7],$ | [8], | A_7 |
| 28. | $Right	ext{-}Arc^n_{\mathtt{AuxK}} \implies ($ | [], | $[0,\ldots,7],$ | [8], | $A_8 = A_7 \cup (0, \mathtt{AuxK}, 8)$) |



| | | λ_1 | λ_2 | β | A |
|-----|-------------------------------------------------------------|-----------------|-------------------|-----------------|-----------------------------------------------|
| 1. | (| [0], | [], | $[1,\ldots,8],$ | Ø) |
| 2. | $Shift^\lambda \Longrightarrow ($ | [0, 1], | [], | $[2,\ldots,8],$ | Ø) |
| 3. | $Right\text{-}Arc^n_{\mathtt{Atr}} \Longrightarrow$ (| [0], | [1], | $[2,\ldots,8],$ | $A_1 = (1, \texttt{Atr}, 2) $ |
| 4. | $Shift^\lambda \implies \ ($ | [0, 1, 2], | [], | $[3,\ldots,8],$ | A_1 |
| 5. | $No	ext{-}Arc^n \implies ($ | [0, 1], | [2], | $[3,\ldots,8],$ | A_1 |
| 6. | $No	ext{-}Arc^n \implies ($ | [0], | [1, 2], | $[3,\ldots,8],$ | A_1 |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \implies ($ | [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3))$ |
| 8. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |
| 9. | $Shift^\lambda \implies \ ($ | $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | A_2 |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies ($ | $[0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | |
| 11. | $Right\text{-}Arc^n_{\mathtt{Sb}} \implies ($ | [0, 1, 2], | [3, 4], | $[5,\ldots,8],$ | $A_4 = A_3 \cup (3, \mathtt{Sb}, 5) \qquad)$ |
| 12. | $No	ext{-}Arc^n \implies ($ | [0, 1], | [2, 3, 4], | $[5,\ldots,8],$ | A_4 |
| 13. | $Left\text{-}Arc^n_{\mathtt{Aux} \mathtt{P}} \implies ($ | [0], | $[1,\ldots,4],$ | $[5,\ldots,8],$ | $A_5 = A_4 \cup (5, \mathtt{AuxP}, 1))$ |
| 14. | $Shift^\lambda \implies \ ($ | $[0,\ldots,5],$ | [], | [6, 7, 8], | A_5 |
| 15. | $No	ext{-}Arc^n \implies ($ | $[0,\ldots,4],$ | [5], | [6, 7, 8], | A_5 |
| 16. | $No\text{-}Arc^n \Longrightarrow ($ | $[0,\ldots,3],$ | [4, 5], | [6, 7, 8], | A_5 |
| 17. | $Right\text{-}Arc^n_{\mathtt{Aux}^{\mathbf{p}}} \implies ($ | [0, 1, 2], | [3, 4, 5], | [6, 7, 8], | $A_6 = A_5 \cup (3, \mathtt{AuxP}, 6))$ |
| 18. | $Shift^\lambda \Longrightarrow ($ | $[0,\ldots,6],$ | [], | [7, 8], | A_6 |
| 19. | $Right	ext{-}Arc^n_{\mathtt{Adv}} \implies ($ | $[0,\ldots,5],$ | [6], | [7, 8], | $A_7 = A_6 \cup (6, Adv, 7) \qquad)$ |
| 20. | $Shift^\lambda \implies \ ($ | $[0,\ldots,7],$ | [], | [8], | A_7 |
| 21. | $No\text{-}Arc^n \Longrightarrow$ (| $[0,\ldots,6],$ | [7], | [8], | A_7 |
| 22. | $No-Arc^n \Longrightarrow ($ | $[0,\ldots,5],$ | [6, 7], | [8], | A_7 |
| 23. | $No-Arc^n \implies ($ | $[0,\ldots,4],$ | [5, 6, 7], | [8], | A_7 |
| 24. | $No-Arc^n \implies ($ | $[0,\ldots,3],$ | $[4,\ldots,7],$ | [8], | A_7 |
| 25. | $No-Arc^n \implies ($ | [0, 1, 2], | $[3, \ldots, 7],$ | [8], | A_7 |
| 26. | $No-Arc^n \implies ($ | [0,1], | $[2,\ldots,7],$ | [8], | A_7 |
| 27. | No-Arc ⁿ \Longrightarrow (| [0], | $[1, \ldots, 7],$ | [8], | A_7) |
| 28. | RIGHT-ARC $_{\text{Aux}K}^n \Longrightarrow ($ | [], | $[0,\ldots,7],$ | [8], | $A_8 = A_7 \cup (0, \mathtt{AuxK}, 8))$ |
| 29. | $SHIFT^\lambda \Longrightarrow ($ | $[0,\ldots,8],$ | [], | [], | A_8 |



| | | λ_1 | λ_2 | β | A | |
|-----|------------------------------------------------------------|------------------|-----------------|-----------------|----------------------------------------|---|
| 1. | (| [0], | [], | $[1,\ldots,8],$ | Ø |) |
| 2. | $Shift^\lambda \Longrightarrow$ (| ([0,1], | [], | $[2,\ldots,8],$ | Ø |) |
| 3. | $Right	ext{-}Arc^n_{\mathtt{Atr}} \implies 0$ | ([0], | [1], | $[2,\ldots,8],$ | $A_1=(1,\mathtt{Atr},2)$ |) |
| 4. | $Shift^\lambda \Longrightarrow$ (| [0,1,2], | [], | $[3,\ldots,8],$ | A_1 |) |
| 5. | $No-Arc^n \implies 0$ | ([0,1], | [2], | $[3,\ldots,8],$ | A_1 |) |
| 6. | $No-Arc^n \implies 0$ | ([0], | [1, 2], | $[3,\ldots,8],$ | A_1 |) |
| 7. | $Right	ext{-}Arc^n_{\mathtt{Pred}} \implies$ | [], | [0, 1, 2], | $[3,\ldots,8],$ | $A_2 = A_1 \cup (0, \texttt{Pred}, 3)$ |) |
| 8. | Shift $^{\lambda} \Longrightarrow$ (| $([0,\ldots,3],$ | [], | $[4,\ldots,8],$ | A_2 |) |
| 9. | $Shift^\lambda \Longrightarrow$ (| $[0,\ldots,4],$ | [], | $[5,\ldots,8],$ | A_2 |) |
| 10. | $Left\text{-}Arc^n_{\mathtt{AuxZ}} \implies 0$ | $([0,\ldots,3],$ | [4], | $[5,\ldots,8],$ | $A_3=A_2\cup(5,\mathtt{AuxZ},4)$ |) |
| 11. | $Right\text{-}Arc^n_{Sb} \Longrightarrow$ | [0,1,2], | [3, 4], | $[5,\ldots,8],$ | $A_4=A_3\cup(3,\operatorname{Sb},5)$ |) |
| 12. | $No\text{-}Arc^n \Longrightarrow$ | ([0,1], | [2, 3, 4], | $[5,\ldots,8],$ | A_4 |) |
| 13. | $Left\text{-}Arc^n_{\mathtt{Aux}\mathtt{P}} \implies 0$ | ([0], | $[1,\ldots,4],$ | $[5,\ldots,8],$ | $A_5 = A_4 \cup (5, \mathtt{AuxP}, 1)$ |) |
| 14. | $Shift^\lambda \Longrightarrow$ (| $([0,\ldots,5],$ | [], | [6, 7, 8], | A_5 |) |
| 15. | $No\text{-}Arc^n \Longrightarrow$ | $[0,\ldots,4],$ | [5], | [6, 7, 8], | A_5 |) |
| 16. | $No\text{-}Arc^n \Longrightarrow$ | $[0,\ldots,3],$ | [4, 5], | [6, 7, 8], | A_5 |) |
| 17. | $Right-Arc^n_{AuxP} \Longrightarrow$ | [0,1,2], | [3, 4, 5], | [6, 7, 8], | $A_6=A_5\cup(3,\mathtt{AuxP},6)$ |) |
| 18. | $Shift^\lambda \Longrightarrow 0$ | $[0,\ldots,6],$ | [], | [7, 8], | A_6 |) |
| 19. | $Right	ext{-}Arc^n_{\mathtt{Adv}} \Longrightarrow$ | $[0,\ldots,5],$ | [6], | [7, 8], | $A_7 = A_6 \cup (6, \mathtt{Adv}, 7)$ |) |
| 20. | Shift $^{\lambda} \Longrightarrow$ (| $([0,\ldots,7],$ | [], | [8], | A_7 |) |
| 21. | $No-Arc^n \implies 0$ | $([0,\ldots,6],$ | [7], | [8], | A_7 |) |
| 22. | $No-Arc^n \implies 0$ | $([0,\ldots,5],$ | [6, 7], | [8], | A_7 |) |
| 23. | $No\text{-}Arc^n \implies 0$ | $[0,\ldots,4],$ | [5, 6, 7], | [8], | A_7 |) |
| 24. | $No\text{-}Arc^n \implies 0$ | $[0,\ldots,3],$ | $[4,\ldots,7],$ | [8], | A_7 |) |
| 25. | $No\text{-}Arc^n \Longrightarrow$ | [0, 1, 2], | $[3,\ldots,7],$ | [8], | A_7 |) |
| 26. | $No\text{-}Arc^n \Longrightarrow$ | ([0,1], | $[2,\ldots,7],$ | [8], | A_7 |) |
| 27. | $No\text{-}Arc^n \Longrightarrow$ | [0], | $[1,\ldots,7],$ | [8], | A_7 |) |
| 28. | $Right\text{-}Arc^n_{\mathtt{Aux}^{\c K}} \Longrightarrow$ | [], | $[0,\ldots,7],$ | [8], | $A_8 = A_7 \cup (0, \mathtt{AuxK}, 8)$ |) |
| 29. | $Shift^\lambda \Longrightarrow$ (| $[0,\ldots,8],$ | [], | [], | A_8 |) |

Transitions LEFT-ARC $_{i}^{p}$ $(\lambda_1|i,\lambda_2,i|\beta,A) \Rightarrow (\lambda_1,[],i|\beta,A\cup\{(i,l,i)\})$ RIGHT-ARC^p $(\lambda_1|i,\lambda_2,j|\beta,A) \Rightarrow (\lambda_1|i|j,[],\beta,A\cup\{(i,l,j)\})$ NO-ARC^p $(\lambda_1|i,\lambda_2,\beta,A) \Rightarrow (\lambda_1,i|\lambda_2,\beta,A)$ $SHIFT^{\lambda}$ $(\lambda_1, \lambda_2, i | \beta, A) \Rightarrow (\lambda_1, \lambda_2 | i, [], \beta, A)$ **Preconditions** LEFT-ARC $_{I}^{p}$ $\neg [i = 0]$ $\neg \exists k \exists l' [(k, l', i) \in A]$ RIGHT-ARC₁^p $\neg \exists k \exists l'[(k, l', j) \in A]$ No-Arc^p $\exists k \exists l[(k,l,i) \in A]$

Figure: Transitions for the arc-eager stack-based parsing algorithm.

The projective, list-based parser uses the same basic strategy as its non-projective counterpart, but skips any pair (i, j) that could give rise to a non-projective dependency arc.

Skipping many node pairs makes it more efficient in practice, although the worst-case time complexity remains the same.

Illustration Transition sequence for projective English example sentence (nearly identical to the arc-eager stack-based sequence):

[ROOT]₀ Economic₁ news₂ had₃ little₄ effect₅ on₆ financial₇ markets₈ .₉

| | λ_1 | λ_2 | β | A | |
|----|-------------|-------------|-----------------|---|---|
| 1. | ([0], | [], | $[1,\ldots,9],$ | Ø |) |

Projective list-based algorithm

Illustration Transition sequence for projective English example sentence (nearly identical to the arc-eager stack-based sequence):

[ROOT]₀ Economic₁ news₂ had₃ little₄ effect₅ on₆ financial₇ markets₈ .₉

| | | | λ_1 | λ_2 | β | A | |
|----|--------------------------|---|-------------|-------------|-----------------|---|---|
| 1. | | (| [0], | [], | $[1,\ldots,9],$ | Ø |) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | [], | $[2,\ldots,9],$ | Ø | |

Illustration Transition sequence for projective English example sentence (nearly identical to the arc-eager stack-based sequence):

-NMOD-[ROOT]₀ Economic₁ news₂ had₃ little₄ effect₅ on₆ financial₇ markets₈ .₉

| | | λ_1 | λ_2 | β | A | |
|----|----------------------------------------------|-------------|-------------|-----------------|-----------------------------------|---|
| 1. | | ([0], | [], | $[1,\ldots,9],$ | Ø |) |
| 2. | $Shift^\lambda \implies$ | ([0,1], | [], | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | ([0], | [], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |

—NMOD— [ROOT]₀ Economic₁ news₂ had₃ little₄ effect₅ on₆ financial₇ markets₈ .₉

| | | | λ_1 | λ_2 | β | | \overline{A} | |
|----|----------------------------------------------|---|-------------|-------------|----------------|------------|-----------------------------------|---|
| 1. | | (| [0], | [], | $[1,\ldots,9]$ | 9], | Ø |) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | [] | $[2,\ldots,9]$ | 9], | Ø |) |
| 3. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0], | [], | $[2,\ldots,9]$ | $\theta],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $Shift^\lambda \implies$ | (| [0, 2], | [], | $[3,\ldots,9]$ | $\theta],$ | A_1 |) |

Illustration Transition sequence for projective English example sentence (nearly identical to the arc-eager stack-based sequence):

NMOD—SBJ [ROOT]₀ Economic₁ news₂ had₃ little₄ effect₅ on₆ financial₇ markets₈ .₉

| | | | λ_1 |) | β | | A | |
|----|----------------------------------------------|---|-------------|---|---------|---------------|-------------------------------------------|---|
| 1. | | (| [0], | | , [1 | $,\ldots,9],$ | Ø |) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | | , [2 | $,\ldots,9],$ | Ø |) |
| 3. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0], | | , [2 | $,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $Shift^\lambda \implies$ | (| [0, 2], | | , [3 | $,\ldots,9],$ | A_1 |) |
| 5. | $Left\text{-}Arc^p_{\mathtt{SBJ}} \implies$ | (| [0], | | , [3 | $,\ldots,9],$ | $A_2 = A_1 \cup \{(3, \mathtt{SBJ}, 2)\}$ |) |

Projective list-based algorithm

| | | | λ_1 | | λ_2 | β | A | |
|----|------------------------------------------------------|---|-------------|---|-------------|-------------------|--------------------------------------------|---|
| 1. | | (| [0], | [|], | $[1,\ldots,9],$ | Ø |) |
| 2. | $Shift^\lambda \Longrightarrow$ | (| [0, 1], | [|], | $[2, \ldots, 9],$ | Ø |) |
| 3. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0], | |], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $Shift^\lambda \Longrightarrow$ | (| [0, 2], | |], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left\text{-}Arc^p_{\mathtt{SBJ}} \implies$ | (| [0], | |], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3, \mathtt{SBJ}, 2)\}$ |) |
| 6. | $Right\text{-}Arc^p_{\mathtt{ROOT}} \Longrightarrow$ | (| [0, 3], | [|], | $[4,\ldots,9],$ | $A_3 = A_2 \cup \{(0, \mathtt{ROOT}, 3)\}$ |) |

| | | | λ_1 | λ | β | | A | |
|----|------------------------------------------------------|---|-------------|----|---------|-----------------|-------------------------------------------|---|
| 1. | | (| [0], | [] | , [1 | $[0,\ldots,9],$ | Ø |) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | | , [2 | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0], | [] | , [2 | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $Shift^\lambda \implies$ | (| [0, 2], | | , [3 | $[8,\ldots,9],$ | A_1 |) |
| 5. | $Left\text{-}Arc^p_{\mathtt{SBJ}} \implies$ | (| [0], | [] | , [3 | $[8,\ldots,9],$ | $A_2 = A_1 \cup \{(3, \mathtt{SBJ}, 2)\}$ |) |
| 6. | $Right\text{-}Arc^p_{\mathtt{ROOT}} \Longrightarrow$ | (| [0, 3], | [] | , [4 | $[1,\ldots,9],$ | $A_3=A_2\cup\{(0,\mathtt{ROOT},3)\}$ |) |
| 7. | $Shift^\lambda \implies$ | (| [0, 3, 4], | | , [5 | $[5,\ldots,9],$ | A_3 |) |

Projective list-based algorithm

| | | | λ_1 | λ_2 | $oldsymbol{eta}$ | A | |
|----|-----------------------------------------------------|---|-------------|-------------|------------------|--------------------------------------------|---|
| 1. | | (| [0], | [], | $[1,\ldots,9],$ | Ø |) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | [] | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0], | [], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $Shift^\lambda \implies$ | (| [0, 2], | [] | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left\text{-}Arc^p_{\mathtt{SBJ}} \implies$ | (| [0], | [], | | $A_2=A_1\cup\{(3,\mathtt{SBJ},2)\}$ |) |
| 6. | $Right	ext{-}Arc^p_{\mathtt{ROOT}} \Longrightarrow$ | (| [0, 3], | [], | $[4,\ldots,9],$ | $A_3=A_2\cup\{(0,\mathtt{ROOT},3)\}$ |) |
| 7. | $Shift^\lambda \implies$ | (| [0, 3, 4], | [], | $[5,\ldots,9],$ | A_3 |) |
| 8. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3], | [], | $[5,\ldots,9],$ | $A_4 = A_3 \cup \{(5, \mathtt{NMOD}, 4)\}$ |) |

| | | | λ_1 | λ_2 | β | A | |
|----|----------------------------------------------|---|-------------|---------------|-------------------|--------------------------------------------|---|
| 1. | | (| [0], | [], | $[1,\ldots,9],$ | Ø |) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | Π, | $[2, \ldots, 9],$ | Ø |) |
| 3. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0], | [], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $Shift^\lambda \Longrightarrow$ | (| [0, 2], | [], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left\text{-}Arc^p_{\mathtt{SBJ}} \implies$ | (| [0], | $[\bar{\ }],$ | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3, \mathtt{SBJ}, 2)\}$ |) |
| 6. | $Right	ext{-}Arc^p_{\mathtt{ROOT}} \implies$ | (| [0, 3], | [], | $[4,\ldots,9],$ | $A_3 = A_2 \cup \{(0, \mathtt{ROOT}, 3)\}$ |) |
| 7. | $Shift^\lambda \implies$ | (| [0, 3, 4], | [], | $[5,\ldots,9],$ | A_3 |) |
| 8. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3], | [], | $[5,\ldots,9],$ | $A_4=A_3\cup\{(5,\mathtt{NMOD},4)\}$ |) |
| 9. | $Right	ext{-}Arc^p_{\mathtt{OBJ}} \implies$ | (| [0, 3, 5], | [], | $[6,\ldots,9],$ | $A_5 = A_4 \cup \{(3, \mathtt{OBJ}, 5)\}$ |) |

| | | | λ_1 | λ_2 | β | A | |
|-----|-----------------------------------------------------|---|---------------|-------------|-----------------|--------------------------------------------|---|
| 1. | | (| [0], | [], | $[1,\ldots,9],$ | Ø |) |
| 2. | $Shift^\lambda \Longrightarrow$ | (| [0, 1], | [], | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0], | [] | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $Shift^\lambda \Longrightarrow$ | (| [0, 2], | [], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left\text{-}Arc^p_{\mathtt{SBJ}} \implies$ | (| [0], | [] | $[3,\ldots,9],$ | $A_2=A_1\cup\{(3,\mathtt{SBJ},2)\}$ |) |
| 6. | $Right	ext{-}Arc^p_{\mathtt{ROOT}} \Longrightarrow$ | (| [0, 3], | [], | $[4,\ldots,9],$ | $A_3 = A_2 \cup \{(0, \mathtt{ROOT}, 3)\}$ |) |
| 7. | $Shift^\lambda \implies$ | (| [0, 3, 4], | [], | $[5,\ldots,9],$ | A_3 |) |
| 8. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | | | [], | | $A_4 = A_3 \cup \{(5, \mathtt{NMOD}, 4)\}$ |) |
| 9. | $Right	ext{-}Arc^p_{\mathtt{OBJ}} \Longrightarrow$ | (| [0, 3, 5], | [], | | $A_5 = A_4 \cup \{(3, \mathtt{OBJ}, 5)\}$ |) |
| 10. | $Right	ext{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [], | [7, 8, 9], | $A_6=A_5\cup\{(5,\mathtt{NMOD},6)\}$ |) |

Non-projective list-based algorithm

Projective list-based algorithm

| | | | λ_1 | λ_2 | β | A | |
|-----|-----------------------------------------------------|---|------------------|-------------|-------------------|--------------------------------------------|---|
| 1. | | (| [0], | [], | $[1,\ldots,9],$ | Ø |) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | [], | $[2, \ldots, 9],$ | Ø |) |
| 3. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0], | [], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $Shift^\lambda \implies$ | (| [0, 2], | [], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left\text{-}Arc^p_{\mathtt{SBJ}} \implies$ | (| [0], | [] | $[3,\ldots,9],$ | $A_2=A_1\cup\{(3,\mathtt{SBJ},2)\}$ |) |
| 6. | $Right	ext{-}Arc^p_{\mathtt{ROOT}} \Longrightarrow$ | (| [0, 3], | [] | $[4,\ldots,9],$ | $A_3 = A_2 \cup \{(0, \mathtt{ROOT}, 3)\}$ |) |
| 7. | $Shift^\lambda \implies$ | (| [0, 3, 4], | [], | $[5,\ldots,9],$ | A_3 |) |
| 8. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3], | [] | $[5,\ldots,9],$ | $A_4 = A_3 \cup \{(5, \mathtt{NMOD}, 4)\}$ |) |
| 9. | $Right	ext{-}Arc^p_{\mathtt{OBJ}} \implies$ | (| [0, 3, 5], | [] | $[6,\ldots,9],$ | $A_5 = A_4 \cup \{(3, \mathtt{OBJ}, 5)\}$ |) |
| 10. | $Right	ext{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [] | [7, 8, 9], | $A_6=A_5\cup\{(5,\mathtt{NMOD},6)\}$ |) |
| 11. | $Shift^\lambda \implies$ | (| [0, 3, 5, 6, 7], | [], | [8, 9], | A_6 |) |

Non-projective list-based algorithm

Projective list-based algorithm

| | 30 | _ | | | | | |
|-----|-----------------------------------------------------|---|------------------|-------------|-------------------|--------------------------------------------|---|
| | | | λ_1 | λ_2 | β | A | |
| 1. | | (| [0], | [], | $[1,\ldots,9],$ | Ø |) |
| 2. | $Shift^\lambda \Longrightarrow$ | (| [0, 1], | [], | $[2, \ldots, 9],$ | Ø |) |
| 3. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0], | [] | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $Shift^\lambda \implies$ | (| [0, 2], | [], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left\text{-}Arc^p_{\mathtt{SBJ}} \implies$ | (| [0], | [], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3,\mathtt{SBJ},2)\}$ |) |
| 6. | $Right	ext{-}Arc^p_{\mathtt{ROOT}} \Longrightarrow$ | (| [0, 3], | [], | $[4,\ldots,9],$ | $A_3 = A_2 \cup \{(0, \mathtt{ROOT}, 3)\}$ |) |
| 7. | $Shift^\lambda \implies$ | (| [0, 3, 4], | [], | $[5,\ldots,9],$ | A_3 |) |
| 8. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3], | [], | $[5,\ldots,9],$ | $A_4 = A_3 \cup \{(5, \mathtt{NMOD}, 4)\}$ |) |
| 9. | $Right	ext{-}Arc^p_{\mathtt{OBJ}} \implies$ | (| [0, 3, 5], | [], | $[6,\ldots,9],$ | $A_5 = A_4 \cup \{(3, \mathtt{OBJ}, 5)\}$ |) |
| 10. | $Right	ext{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [], | [7, 8, 9], | $A_6 = A_5 \cup \{(5, \mathtt{NMOD}, 6)\}$ |) |
| 11. | $Shift^\lambda \implies$ | (| [0, 3, 5, 6, 7], | [], | [8, 9], | A_6 |) |
| 12. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [], | [8, 9], | $A_7 = A_6 \cup \{(8, \mathtt{NMOD}, 7)\}$ |) |

Non-projective list-based algorithm

Projective list-based algorithm

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|-----|----------------------------------------------|---|------------------|-------------|-------------------|-----------------------------------------------|
| | | | λ_1 | λ_2 | β | A |
| 1. | | (| [0], | [], | $[1,\ldots,9],$ | Ø) |
| 2. | $Shift^\lambda \Longrightarrow$ | (| [0,1], | [], | $[2, \ldots, 9],$ | \varnothing |
| 3. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0], | [], | $[2,\ldots,9],$ | $A_1 = \{(2, \texttt{NMOD}, 1)\} $ |
| 4. | $Shift^\lambda \Longrightarrow$ | (| [0, 2], | [] | $[3,\ldots,9],$ | A_1) |
| 5. | $Left\text{-}Arc^p_{\mathtt{SBJ}} \implies$ | (| [0], | [], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3, SBJ, 2)\}$ |
| 6. | $Right	ext{-}Arc^p_{\mathtt{ROOT}} \implies$ | (| [0, 3], | [], | $[4,\ldots,9],$ | $A_3 = A_2 \cup \{(0, \mathtt{ROOT}, 3)\}$ |
| 7. | $Shift^\lambda \implies$ | (| [0, 3, 4], | [], | $[5,\ldots,9],$ | A_3 |
| 8. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0,3], | [], | $[5,\ldots,9],$ | $A_4 = A_3 \cup \{(5, \mathtt{NMOD}, 4)\})$ |
| 9. | $Right	ext{-}Arc^p_{OBJ} \implies$ | (| [0, 3, 5], | [], | $[6,\ldots,9],$ | $A_5 = A_4 \cup \{(3, \mathtt{OBJ}, 5)\} $ |
| 10. | $Right	ext{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [], | [7, 8, 9], | $A_6 = A_5 \cup \{(5, \texttt{NMOD}, 6)\})$ |
| 11. | $Shift^\lambda \implies$ | (| [0, 3, 5, 6, 7], | [] | [8, 9], | A_6 |
| 12. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [], | [8, 9], | $A_7 = A_6 \cup \{(8, \texttt{NMOD}, 7)\} $ |
| 13. | $Right	ext{-}Arc^p_{\mathtt{PMOD}} \implies$ | (| [0, 3, 5, 6, 8], | [], | [9], | $A_8 = A_7 \cup \{(6, PMOD, 8)\}$) |

| | | λ_1 | λ_2 | eta | A |
|-----|-----------------------------------------------------|-------------|-------------|-------------------|-----------------------------------------------|
| 1. | | ([0], | [], | $[1,\ldots,9],$ | Ø) |
| 2. | $Shift^\lambda \Longrightarrow$ | ([0,1], | [], | $[2, \ldots, 9],$ | Ø) |
| 3. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | ([0], | [], | $[2,\ldots,9],$ | $A_1 = \{(2, \texttt{NMOD}, 1)\}$ |
| 4. | $Shift^\lambda \Longrightarrow$ | ([0,2], | [], | $[3,\ldots,9],$ | A_1 |
| 5. | $Left\text{-}Arc^p_{\mathtt{SBJ}} \implies$ | ([0], | [], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3, SBJ, 2)\}$ |
| 6. | $Right	ext{-}Arc^p_{\mathtt{ROOT}} \Longrightarrow$ | ([0,3], | [], | $[4,\ldots,9],$ | $A_3 = A_2 \cup \{(0, \mathtt{ROOT}, 3)\}$ |
| 7. | $Shift^\lambda \Longrightarrow$ | ([0,3,4] | , [], | $[5,\ldots,9],$ | A_3 |
| 8. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | ([0,3], | [], | $[5,\ldots,9],$ | $A_4 = A_3 \cup \{(5, NMOD, 4)\}$ |
| 9. | $Right\text{-}Arc^p_{OBJ} \implies$ | ([0,3,5] | , $[]$, | $[6,\ldots,9],$ | $A_5 = A_4 \cup \{(3, OBJ, 5)\} $ |
| 10. | $Right	ext{-}Arc^p_{\mathtt{NMOD}} \Longrightarrow$ | ([0,3,5, | [6], | [7, 8, 9], | $A_6 = A_5 \cup \{(5, \texttt{NMOD}, 6)\})$ |
| 11. | $Shift^\lambda \implies$ | ([0,3,5, | [6,7], [], | [8, 9], | A_6 |
| 12. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | ([0,3,5, | 6], [], | [8, 9], | $A_7 = A_6 \cup \{(8, NMOD, 7)\}$ |
| 13. | $Right	ext{-}Arc^p_{\mathtt{PMOD}} \implies$ | ([0,3,5, | [6, 8], [], | [9], | $A_8 = A_7 \cup \{(6, PMOD, 8)\}$ |
| 14. | $No	ext{-}Arc^p \implies$ | ([0,3,5, | 6], [8], | [9], | A_8 |

Projective list-based algorithm

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|-----|----------------------------------------------|-----|-------------------------------------|-------------|------------------------------------|-----------------------------------------------|
| | | | λ_1 | λ_2 | β | A |
| 1. | | (| [0], | [], | $[1,\ldots,9],$ | Ø) |
| 2. | $Shift^\lambda \Longrightarrow$ | (| [0,1], | [], | $[2, \ldots, 9],$ | \varnothing) |
| 3. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0], | [], | $[2,\ldots,9],$ | $A_1 = \{(2, \texttt{NMOD}, 1)\}$ |
| 4. | $Shift^\lambda \implies$ | (| [0, 2], | [], | $[3,\ldots,9],$ | A_1 |
| 5. | $Left\text{-}Arc^p_{\mathtt{SBJ}} \implies$ | (| [0], | [], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3, SBJ, 2)\}$ |
| 6. | $Right	ext{-}Arc^p_{\mathtt{ROOT}} \implies$ | (| [0, 3], | [], | $[4,\ldots,9],$ | $A_3 = A_2 \cup \{(0, \mathtt{ROOT}, 3)\}$) |
| 7. | $Shift^\lambda \implies$ | (| [0, 3, 4], | [], | $[5,\ldots,9],$ | A_3 |
| 8. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3], | [], | $[5,\ldots,9],$ | $A_4 = A_3 \cup \{(5, \mathtt{NMOD}, 4)\})$ |
| 9. | $Right	ext{-}Arc^p_{\mathtt{OBJ}} \implies$ | (| [0, 3, 5], | [], | $[6,\ldots,9],$ | $A_5 = A_4 \cup \{(3, OBJ, 5)\} $ |
| 10. | $Right	ext{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [], | [7, 8, 9], | $A_6 = A_5 \cup \{(5, \texttt{NMOD}, 6)\})$ |
| 11. | $Shift^\lambda \implies$ | (| [0, 3, 5, 6, 7], | [], | [8, 9], | A_6 |
| 12. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [], | [8, 9], | $A_7 = A_6 \cup \{(8, \texttt{NMOD}, 7)\})$ |
| 13. | $Right	ext{-}Arc^p_{\mathtt{PMOD}} \implies$ | (| [0, 3, 5, 6, 8], | [], | [9], | $A_8 = A_7 \cup \{(6, \texttt{PMOD}, 8)\})$ |
| 14. | $No	ext{-}Arc^p \implies$ | (| [0, 3, 5, 6], | [8], | [9], | A_8 |
| 15. | $No	ext{-}arc^p \implies$ | (| [0, 3, 5], | [6, 8], | [9], | A_8 |

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|-----|-----------------------------------------------------|---|-------------------------------------|-----------------------|-------------------|-----------------------------------------------|
| | | | λ_1 | λ_2 | β | A |
| 1. | | (| [0], | [], | $[1,\ldots,9],$ | Ø) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | [], | $[2, \ldots, 9],$ | Ø) |
| 3. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0], | [], | $[2,\ldots,9],$ | $A_1 = \{(2, \texttt{NMOD}, 1)\} $ |
| 4. | $Shift^\lambda \Longrightarrow$ | (| [0, 2], | [], | $[3, \ldots, 9],$ | A_1 |
| 5. | $Left\text{-}Arc^p_{\mathtt{SBJ}} \implies$ | (| [0], | [] | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3, SBJ, 2)\}$ |
| 6. | $Right	ext{-}Arc^p_{\mathtt{ROOT}} \Longrightarrow$ | (| [0, 3], | [], | $[4,\ldots,9],$ | $A_3 = A_2 \cup \{(0, ROOT, 3)\}$ |
| 7. | $Shift^\lambda \Longrightarrow$ | (| [0, 3, 4], | [], | $[5,\ldots,9],$ | A_3 |
| 8. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0,3], | [], | $[5,\ldots,9],$ | $A_4 = A_3 \cup \{(5, \texttt{NMOD}, 4)\})$ |
| 9. | $Right	ext{-}Arc^p_{\mathtt{OBJ}} \implies$ | (| [0, 3, 5], | [], | $[6,\ldots,9],$ | $A_5 = A_4 \cup \{(3, \mathtt{OBJ}, 5)\} $ |
| 10. | $Right	ext{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [], | [7, 8, 9], | $A_6 = A_5 \cup \{(5, \texttt{NMOD}, 6)\})$ |
| 11. | Shift $^{\lambda} \Longrightarrow$ | (| [0, 3, 5, 6, 7], | [], | [8, 9], | A_6 |
| 12. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [], | [8, 9], | $A_7 = A_6 \cup \{(8, \texttt{NMOD}, 7)\})$ |
| 13. | $Right	ext{-}Arc^p_{\mathtt{PMOD}} \implies$ | (| [0, 3, 5, 6, 8], | [], | [9], | $A_8 = A_7 \cup \{(6, \texttt{PMOD}, 8)\})$ |
| 14. | $No	ext{-}Arc^p \implies$ | (| [0, 3, 5, 6], | [8], | [9], | A_8 |
| 15. | $No	ext{-}arc^p \implies$ | (| [0, 3, 5], | [6, 8], | [9], | A_8 |
| 16. | $No	ext{-}arc^p \implies$ | (| [0, 3], | [5, 6, 8], | [9], | A_8 |

| | | | λ_1 | λ_2 | β | A | |
|-----|------------------------------------------------------|---|------------------|-------------|-----------------|--------------------------------------------|---|
| 1. | (| (| [0], | [], | $[1,\ldots,9],$ | Ø |) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | [], | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0], | [], | $[2,\ldots,9],$ | $A_1 = \{(2, \texttt{NMOD}, 1)\}$ |) |
| 4. | $Shift^\lambda \Longrightarrow$ | (| [0, 2], | [], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left\text{-}Arc^p_{\mathtt{SBJ}} \implies$ | (| [0], | [], | $[3,\ldots,9],$ | $A_2 = A_1 \cup \{(3,\mathtt{SBJ},2)\}$ |) |
| 6. | $Right\text{-}Arc^p_{\mathtt{ROOT}} \Longrightarrow$ | (| [0, 3], | [], | $[4,\ldots,9],$ | $A_3 = A_2 \cup \{(0, \mathtt{ROOT}, 3)\}$ |) |
| 7. | $Shift^\lambda \implies$ | (| [0, 3, 4], | [], | $[5,\ldots,9],$ | A_3 |) |
| 8. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3], | [], | $[5,\ldots,9],$ | $A_4 = A_3 \cup \{(5, \texttt{NMOD}, 4)\}$ |) |
| 9. | $Right	ext{-}Arc^p_{\mathtt{OBJ}} \implies$ | (| [0, 3, 5], | [], | $[6,\ldots,9],$ | $A_5 = A_4 \cup \{(3, \mathtt{OBJ}, 5)\}$ |) |
| 10. | $Right	ext{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [], | [7, 8, 9], | $A_6 = A_5 \cup \{(5, \mathtt{NMOD}, 6)\}$ |) |
| 11. | $Shift^\lambda \implies$ | (| [0, 3, 5, 6, 7], | [], | [8, 9], | A_6 |) |
| 12. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [], | [8, 9], | $A_7 = A_6 \cup \{(8, \mathtt{NMOD}, 7)\}$ |) |
| 13. | $Right	ext{-}Arc^p_{\mathtt{PMOD}} \implies$ | (| [0, 3, 5, 6, 8], | [], | [9], | $A_8 = A_7 \cup \{(6, \texttt{PMOD}, 8)\}$ |) |
| 14. | $No	ext{-}Arc^p \implies$ | (| [0, 3, 5, 6], | [8], | [9], | A_8 |) |
| 15. | $No	ext{-}arc^p \implies$ | (| [0, 3, 5], | [6, 8], | [9], | A_8 |) |
| 16. | $No	ext{-}arc^p \implies$ | (| [0, 3], | [5, 6, 8], | [9], | A_8 |) |
| 17. | $Right	ext{-}Arc^p_{\mathtt{P}} \implies$ | (| [0, 3, 9], | [], | [], | $A_9 = A_8 \cup \{(3, P, 9)\}$ |) |

| | | | λ_1 | λ_2 | eta | A | |
|-----|-----------------------------------------------------|---|------------------|-------------|-----------------|--------------------------------------------|---|
| 1. | | (| [0], | [], | $[1,\ldots,9],$ | Ø |) |
| 2. | $Shift^\lambda \implies$ | (| [0, 1], | [], | $[2,\ldots,9],$ | Ø |) |
| 3. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0], | [], | $[2,\ldots,9],$ | $A_1 = \{(2, \mathtt{NMOD}, 1)\}$ |) |
| 4. | $Shift^\lambda \implies$ | (| [0, 2], | [], | $[3,\ldots,9],$ | A_1 |) |
| 5. | $Left\text{-}Arc^p_{\mathtt{SBJ}} \implies$ | (| [0], | [], | $[3,\ldots,9],$ | $A_2=A_1\cup\{(3,\mathtt{SBJ},2)\}$ |) |
| 6. | $Right	ext{-}Arc^p_{\mathtt{ROOT}} \Longrightarrow$ | (| [0, 3], | [], | $[4,\ldots,9],$ | $A_3=A_2\cup\{(0,\mathtt{ROOT},3)\}$ |) |
| 7. | $Shift^\lambda \implies$ | (| [0, 3, 4], | [], | $[5,\ldots,9],$ | A_3 |) |
| 8. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3], | [], | $[5,\ldots,9],$ | $A_4 = A_3 \cup \{(5, \mathtt{NMOD}, 4)\}$ |) |
| 9. | Right-Arc $_{\mathtt{OBJ}}^{p} \Longrightarrow$ | (| [0, 3, 5], | [], | $[6,\ldots,9],$ | |) |
| 10. | $Right	ext{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [], | [7, 8, 9], | $A_6=A_5\cup\{(5,\mathtt{NMOD},6)\}$ |) |
| 11. | $Shift^\lambda \implies$ | (| [0, 3, 5, 6, 7], | [], | [8, 9], | A_6 |) |
| 12. | $Left\text{-}Arc^p_{\mathtt{NMOD}} \implies$ | (| [0, 3, 5, 6], | [], | [8, 9], | $A_7 = A_6 \cup \{(8, \mathtt{NMOD}, 7)\}$ |) |
| 13. | $Right	ext{-}Arc^p_{\mathtt{PMOD}} \implies$ | (| [0, 3, 5, 6, 8], | [], | [9], | $A_8 = A_7 \cup \{(6, \texttt{PMOD}, 8)\}$ |) |
| 14. | $No	ext{-}Arc^p \implies$ | (| [0, 3, 5, 6], | [8], | [9], | A_8 |) |
| 15. | | (| [0, 3, 5], | [6, 8], | [9], | A_8 |) |
| 16. | $No\text{-}arc^p \Longrightarrow$ | (| [0,3], | [5, 6, 8], | [9], | A_8 |) |
| 17. | $Right\text{-}Arc^p_{\mathtt{P}} \implies$ | (| [0, 3, 9], | [], | [], | $A_9 = A_8 \cup \{(3, P, 9)\}$ |) |

Evaluation of the four algorithms in deterministic data-driven parsing: Use an oracle approximated by a classifier trained on treebank data to analyze of the accuracy and efficiency of these systems.

■ Data: CoNLL-X shared task multilingual dependency parsing

Evaluation of the four algorithms in deterministic data-driven parsing: Use an oracle approximated by a classifier trained on treebank data to analyze of the accuracy and efficiency of these systems.

■ Data: CoNLL-X shared task multilingual dependency parsing

Data sets. Tok = number of tokens (\times 1000); Sen = number of sentences (\times 1000); T/S = tokens per sentence (mean); Lem = lemmatization present; CPoS = number of coarse-grained part-of-speech tags; PoS = number of (fine-grained) part-of-speech tags; MSF = number of morphosyntactic features (split into atoms); Dep = number of dependency types; NPT = proportion of non-projective dependencies/tokens (%); NPS = proportion of non-projective dependency graphs/sentences (%).

| Language | Tok | Sen | T/S | Lem | CPoS | PoS | MSF | Dep | NPT | NPS |
|------------|-------|------|------|-----|------|-----|-----|-----|-----|------|
| Arabic | 54 | 1.5 | 37.2 | yes | 14 | 19 | 19 | 27 | 0.4 | 11.2 |
| Bulgarian | 190 | 14.4 | 14.8 | no | 11 | 53 | 50 | 18 | 0.4 | 5.4 |
| Chinese | 337 | 57.0 | 5.9 | no | 22 | 303 | 0 | 82 | 0.0 | 0.0 |
| Czech | 1,249 | 72.7 | 17.2 | yes | 12 | 63 | 61 | 78 | 1.9 | 23.2 |
| Danish | 94 | 5.2 | 18.2 | no | 10 | 24 | 47 | 52 | 1.0 | 15.6 |
| Dutch | 195 | 13.3 | 14.6 | yes | 13 | 302 | 81 | 26 | 5.4 | 36.4 |
| German | 700 | 39.2 | 17.8 | no | 52 | 52 | 0 | 46 | 2.3 | 27.8 |
| Japanese | 151 | 17.0 | 8.9 | no | 20 | 77 | 0 | 7 | 1.1 | 5.3 |
| Portuguese | 207 | 9.1 | 22.8 | yes | 15 | 21 | 146 | 55 | 1.3 | 18.9 |
| Slovene | 29 | 1.5 | 18.7 | yes | 11 | 28 | 51 | 25 | 1.9 | 22.2 |
| Spanish | 89 | 3.3 | 27.0 | yes | 15 | 38 | 33 | 21 | 0.1 | 1.7 |
| Swedish | 191 | 11.0 | 17.3 | no | 37 | 37 | 0 | 56 | 1.0 | 9.8 |
| Turkish | 58 | 5.0 | 11.5 | yes | 14 | 30 | 82 | 25 | 1.5 | 11.6 |

Figure: Data sets

Learning and parsing time for seven parsers on six languages, measured in seconds. NP-L = non-projective list-based; P-L = projective list-based; PP-L = pseudo-projective list-based; P-E = projective arc-eager stack-based; PP-E = pseudo-projective arc-eager stack-based; P-S = projective arc-standard stack-based; PP-S = pseudo-projective arc-standard stack-based.

| | Learning Time | | | | | | | | | |
|--------------|---------------|---------|---------|---------|---------|---------|---------|--|--|--|
| Language | NP-L | P-L | PP-L | P-E | PP-E | P-S | PP-S | | | |
| Arabic | 1,814 | 614 | 603 | 650 | 647 | 1,639 | 1,636 | | | |
| Bulgarian | 6,796 | 2,918 | 2,926 | 2,919 | 2,939 | 3,321 | 3,391 | | | |
| Chinese | 17,034 | 13,019 | 13,019 | 13,029 | 13,029 | 13,705 | 13,705 | | | |
| Czech | 546,880 | 250,560 | 248,511 | 279,586 | 280,069 | 407,673 | 406,857 | | | |
| Danish | 2,964 | 1,248 | 1,260 | 1,246 | 1,262 | 643 | 647 | | | |
| Dutch | 7,701 | 3,039 | 2,966 | 3,055 | 2,965 | 7,000 | 6,812 | | | |
| German | 48,699 | 16,874 | 17,600 | 16,899 | 17,601 | 24,402 | 24,705 | | | |
| Japanese | 211 | 191 | 188 | 203 | 208 | 199 | 199 | | | |
| Portuguese | 25,621 | 8,433 | 8,336 | 8,436 | 8,335 | 7,724 | 7,731 | | | |
| Slovene | 167 | 78 | 90 | 93 | 99 | 86 | 90 | | | |
| Spanish | 1,999 | 562 | 566 | 565 | 565 | 960 | 959 | | | |
| Swedish | 2,410 | 942 | 1,020 | 945 | 1,022 | 1,350 | 1,402 | | | |
| Turkish | 720 | 498 | 519 | 504 | 516 | 515 | 527 | | | |
| Average | 105,713 | 46,849 | 46,616 | 51,695 | 51,876 | 74,798 | 74,691 | | | |
| Parsing Time | | | | | | | | | | |

| | Parsing 11me | | | | | | | | |
|------------|--------------|-------|-------|-------|-------|-------|-------|--|--|
| Language | NP-L | P-L | PP-L | P-E | PP-E | P-S | PP-S | | |
| Arabic | 213 | 103 | 131 | 108 | 135 | 196 | 243 | | |
| Bulgarian | 139 | 93 | 102 | 93 | 103 | 135 | 147 | | |
| Chinese | 1,008 | 855 | 855 | 855 | 855 | 803 | 803 | | |
| Czech | 5,244 | 3,043 | 5,889 | 3,460 | 6,701 | 3,874 | 7,437 | | |
| Danish | 109 | 66 | 83 | 66 | 83 | 82 | 106 | | |
| Dutch | 349 | 209 | 362 | 211 | 363 | 253 | 405 | | |
| German | 781 | 456 | 947 | 455 | 945 | 494 | 1,004 | | |
| Japanese | 10 | 8 | 8 | 9 | 10 | 7 | 7 | | |
| Portuguese | 670 | 298 | 494 | 298 | 493 | 437 | 717 | | |
| Slovene | 69 | 44 | 62 | 47 | 65 | 43 | 64 | | |
| Spanish | 133 | 67 | 75 | 67 | 75 | 80 | 91 | | |
| Swedish | 286 | 202 | 391 | 201 | 391 | 242 | 456 | | |
| Turkish | 218 | 162 | 398 | 162 | 403 | 153 | 380 | | |
| Average | 1,240 | 712 | 1,361 | 782 | 1,496 | 897 | 1,688 | | |

Figure: Parsing efficiency

Nivre, Joakim (Dec. 1, 2008). "Algorithms for Deterministic Incremental Dependency Parsing". In: Computational Linguistics 34.4, pp. 513–553. DOI: 10.1162/coli.07-056-R1-07-027.